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Preface

Welcome to the 3rd International E-Conference on Mathematical Development and Applications (ICOMAA-2020) we organized the third. The aim of our conferences is to bring together scientists and young researchers from all over the world and their work on the fields of mathematics and mathematics, to exchange ideas, to collaborate and to add new ideas to mathematics in a discussion environment. With this interaction, functional analysis, approach theory, differential equations and partial differential equations and the results of applications in the field of Mathematics Education are discussed with our valuable academics, and in mathematical developments both science and young researchers are opened. We are happe to host many prominent experts from different countries who will present the state-of-the-art in real analysis, complex analysis, harmonic and non-harmonic analysis, operator theory and spectral analysis, applied analysis.

However, this year we had to hold our conference online due to the Covid-19 pandemic of the world. Although there are minor faults due to being the first, the satisfaction and positive feedback of our participants gave us strength. I would like to thank first to my team and then to all our participants.

The conference brings together about 190 participants from 20 countries (Algeria, Azerbaijan, Canada, Colombia, Czech Republic, Egypt, Finland, Germany, Indonesia, India, Italy, Kyrgyzstan, Malaysia, Morocco, Pakistan, Saudi Arabia, Thailand, Turkey, United Arab Emirates, USA) and 12 invited talks.

The scientific committee members of ICOMAA-2020 and the external reviewers invested significant time in analyzing and assessing multiple papers, consequently, they hold and maintain a high standard of quality for this conference. The scientific program of the conference features invited talks, followed by contributed oral and poster presentations in seven parallel sessions.

The conference program represents the efforts of many people. I would like to express my gratitude to all members of the scientific committee, external reviewers, sponsors and, honorary committee for their continued support to the ICOMAA. I also thank the invited speakers for presenting their talks on current researches. Also, the success of ICOMAA depends on the effort and talent of researchers in mathematics and its applications that have written and submitted papers on a variety of topics. So, I would like to sincerely thank all participants of ICOMAA-2020 for contributing to this great meeting in many different ways. I believe and hope that each of you will get the maximum benefit from the conference.

Assoc. Prof. Dr. Yusuf ZEREN
Chairman
On behalf of the Organizing Committee

Editor in Chief

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ICOMAA

INTERNATIONAL CONFERENCE on MATHEMATICAL ADVANCES and APPLICATIONS

3rd INTERNATIONAL E-CONFERENCE ON MATHEMATICAL ADVANCES AND APPLICATIONS

Abstract Book

24-27 JUNE, ISTANBUL online video conferencing

Editors

Yusuf Zeren - Necip SIMSEK
Bilal Bilalov

website

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JUNE, 24-27, 2020, ISTANBUL / TURKEY

Abstract Book

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FOREWORDS

Dear Conference Participant,

Welcome to the 3rd International E-Conference on Mathematical Development and Applications (ICOMAA-2020) we organized the third. The aim of our conferences is to bring together scientists and young researchers from all over the world and their work on the fields of mathematics and mathematics, to exchange ideas, to collaborate and to add new ideas to mathematics in a discussion environment. With this interaction, functional analysis, approach theory, differential equations and partial differential equations and the results of applications in the field of Mathematics Education are discussed with our valuable academics, and in mathematical developments both science and young researchers are opened. We are happe to host many prominent experts from different countries who will present the state-of-the-art in real analysis, complex analysis, harmonic and non-harmonic analysis, operator theory and spectral analysis, applied analysis.

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More than 50% of our participants participated from abroad. This shows that the conference meets the criteria of being international.

It is also an aim of the conference to encourage opportunities for collaboration and networking between senior academics and graduate students to advance their new perspective. Additional emphasis on ICOMAA-2020 applies to other areas of science, such as natural sciences, economics, computer science, and various engineering sciences, as well as applications in related fields. The articles submitted to this conference will be addressed on the conference web sites and in the journals listed below:

- Miskolc Mathematical Notes,
- Azerbaijan Journal of Mathematics,
- Sigma Journal of Engineering and Natural Sciences,
- Istanbul Commerce University Journal of Sciences,
- Transactions Issue Mathematics.

This booklet contains the titles and abstracts of almost all invited and contributed talks at the **3rd International E-Conference on Mathematical Advances and Applications**. Only some abstracts were not available at the time of printing the booklet. They will be made available on the conference website http://icomaa2020.com/ when the organizers receive them.

We wish everyone a fruitful conference and pleasant memories throughout the online conference.

Assoc. Prof. Yusuf ZEREN
On Behalf of Organizing Committee
Chairman

It was a big excitement moment when Assoc. Prof. Yusuf ZEREN discussed with me on the issue of "3rd International Mathematical Developments and Applications Conference" (ICOMAA-2020) in Yıldız Technical University, Istanbul. It is a great pleasure that this conference is going to take place now. As one of the organizers of the conference, I am delighted with all the delegates, distinguished mathematicians, speakers and young researchers in this international event. It is expected that delegates and participants will benefit from this conference experience and the legacy of information dissemination will continue.

I wish all of you to have a nice and enjoyable participation in the conference.

Prof. Necip SIMSEK

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INVITED TALKS

Weighted inequalities for discrete iterated Hardy operators

Amiran Gogatishvili[1]
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Abstract

We characterize a three-weight inequality for an iterated discrete Hardy-type operator. In the case when the domain space is a weighted space l^p with $p \in (0,1]$, we develop characterizations which enable us to reduce the problem to another one with p=1. This, in turn, makes it possible to establish an equivalence of the weighted discrete inequality to an~appropriate inequality for iterated Hardy-type operators acting on measurable functions defined on R, for all cases of involved positive exponents.

Keywords: Weighted discrete inequality; supremum operator; iterated operator

References:
A.Gogatishvili, M.Krepela, R.Olhava and L.Pick. Weighted inequalities for discrete iterated Hardy, Mediterr. J. Math. 17(2020), doi 10.1007/s00009-020-01526-2

Hardy Banach Spaces, Cauchy Formula and Riesz Theorem

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[2]*Ganja State University, Ganja, Azerbaijan*

Abstract

In this paper Banach function space and the Hardy classes of analytic functions generated by it are considered. An analogue of the classical Riesz theorem in these classes and the validity of the Cauchy formula for analytic functions from these classes are established. The basicity of parts of system of exponents in the corresponding Hardy classes is proved.

Keywords: Hardy Banach spaces, Riesz theorem, Cauchy formula

References

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Norm inequalities for linear and multilinear singular integrals on weighted and variable exponent Hardy spaces

David Cruz-Uribe
The University of Alabama, USA

Abstract

I will discuss recent work with Kabe Moen and Hanh Nguyen on norm inequalities of the form

$$T: H^{p1}(w_1) \times H^{p2}(w_2) \rightarrow L^p(w),$$

where T is a bilinear Calderon-Zygmund singular integral operator, $0 < p, p_1, p_2 < \infty$ and

$$\frac{1}{p_1} + \frac{1}{p_2} = \frac{1}{p}$$

the weights w, w_1, w_2 are Muckenhoupt weights, and the spaces $H^{p_i}(w_i)$ are the weighted Hardy spaces introduced by Stromberg and Torchinsky. We also consider norm inequalities of the form

$$T: H^{p_1}(\cdot) \times H^{p_2}(\cdot) \rightarrow L^p(\cdot),$$

where $L^p(\cdot)$, is a variable Lebesgue space (intuitively, a classical Lebesgue space with the constant exponent p replaced by an exponent function $p(\cdot)$) and the spaces $H^{p_i}(.)$ are the corresponding variable exponent Hardy spaces, introduced by me and Li-An Wang and independently by Nakai and Sawano. To illustrate our approach we will consider the special case of linear singular integrals. Our proofs, which are simpler than existing proofs, rely heavily on three things: finite atomic decompositions, vector-valued inequalities, and the theory of Rubio de Francia extrapolation.

On Harnack's inequality for some class of non-uniformly degenerated elliptic equations

Dumitru Baleanu[1,2]
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[2]*Institute of Space Sciences, Magurele-Bucharest, Romania*

Abstract

In my talk I am going to present some new trends within the Fractional Mathematical Biology. Some illustrative example will be given.

On local properties of degenerated parabolic equations

Farman Mamedov[1] *and Yusuf Zeren*[2]
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Abstract

In the literature, the termin of local properties is used usually to refer the Holder regularity, Harnack's inequality, two side estimates of fundamental solution and etc. results for 2-nd order parabolic equations, also for the cases of its degenerate and quasilinear analogues (see, e.g. [1], [2]). This abstract relates to the equation

$$\frac{\partial}{\partial x_j}\left(a_{ij}(t,x)\frac{\partial u}{\partial x_i}\right)-\frac{\partial u}{\partial t}=0 \quad (1)$$

with the uniform degeneratacy condition

$$\frac{1}{C}\omega(t,x)|\xi|^2 a_{ij}(t,x)\xi_i\xi_j \le C\omega(t,x)|\xi|^2 \quad (2)$$

for $C>1,\ \forall\xi\in\Re^n,(t,x)\in D$, and D be a bounded domain in half-space $\{t<t_0\}$.

Concerning the function $\omega(t,x)$ to be a measurable positive function and some Muckenhoupt's condition all over special cylinders and some additional assumptions are assumed in order to get the following results.

Theorem. *Positive weak solutions of (1) satisfy the Harnack inequality. Weak solutions of (1) are of Holder continuous.*

Keywords: regularity of solutions, Holder regularity, Harnack's inequality, fundamental solutions, weak solutions, qualitative properties of solutions, a prior estimates.

References:

1. E.M. Landis, Second Order Equations of Elliptic and Parabolic Type, Transl. Math. Monogr. Vol 171, Amer.Math. Soc. Providence RI, 1998.

2. E. DiBenedetto, Degenerate Parabolic Equations, Springer-Verlag, New York, 1993.

On Some Recent Advances about the Convergence of Sequences of Positive Linear Operators and Functionals

Francesco Altomare
Department of Mathematics, University of Bari, Italy
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Abstract

The talk will be devoted to present some new results concerning the convergence of sequences of positive linear operators and functionals in the framework of spaces of bounded functions which are continuous on a given subset of their domain. Among other things some applications will be discussed which are relate to the asymptotic behaviour of the integrals of means as well as the iterates of positive linear operators.
Most of the results extend previous ones contained in [1] and [2, Section 1.3].

Keywords: Positive linear operator, positive linear functional, Bernstein-type operator, iterates of positive operator, integral of means.

References:

[1] F. Altomare, On some convergence criteria for nets of positive linear operators, J. Math. Anal. Appl. 398, 542-552 (2013).

[2] F. Altomare, M. Cappelletti Montano, V. Leonessa and I. Rasa, Markov Operators, Positive Semigroups and Approximation Processes, de Gruyter Studies in Mathematics, 61, Walter de Gruyter GmbH, Berlin/Munich/Boston, 2014.

The s-numbers, Eigenvalues and Generalized trigonometric functions

Jan Lang[1]
[1]*Department of Mathematics, The Ohio State University,*
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Abstract

The main theme of this talk is to discuss behavior of s-numbers for integral operators of Hardy type and related Sobolev embeddings, together with eigenvalues for corresponding non-linear problems. Generalized trigonometric functions, which were first introduced by Lundberg 1879, will play a crucial role and some of their interesting properties will be discussed.

Elliptic equations with degenerate weights

Lars Diening
Bielefeld University

Abstract

We study the regularity of elliptic equations with degenerate elliptic weights in the linear case

$$-\,\boldsymbol{div}(\boldsymbol{A}(\boldsymbol{x})\boldsymbol{\nabla u}) \;=\; -\,\boldsymbol{div}(\boldsymbol{A}(\boldsymbol{x})\boldsymbol{G}),$$

as well as in the non-linear case

$$-\,\boldsymbol{div}\,(|\boldsymbol{M}(\boldsymbol{x})\boldsymbol{\nabla u}|^{p-2}\boldsymbol{M}^{2}(\boldsymbol{x})\boldsymbol{\nabla u}) \;=\; -\,\boldsymbol{div}\,(|\boldsymbol{M}(\boldsymbol{x})\boldsymbol{G}|^{p-2}\boldsymbol{M}^{2}(\boldsymbol{x})\boldsymbol{G}),$$

where $\mathbf{1} < \boldsymbol{p} < \infty$ and $\boldsymbol{G}$ is the given data. The mappings $\boldsymbol{A}, \boldsymbol{M} : \boldsymbol{\Omega} \to \boldsymbol{R}^{n\times n}_{(sym)}$ are symmetric, positive definite, matrix-valued weights, which may be degenerate. This includes for examples simple weights as $|\boldsymbol{x}|^{\pm\epsilon}$Id. We establish a novel condition on the weight $\boldsymbol{M}$. Instead of a BMO (bounded mean oscillation) smallness condition for $\boldsymbol{M}$, we use a BMO smallness condition on its logarithm log $\boldsymbol{M}$, which is new even for the linear case. Under this condition we show that local higher integrability of $\boldsymbol{G}$ transfers to ∇u. The sharpness of our estimates is proved by examples. The talk is based on joint work with Anna Balci, Raffaella Giova and Antonia Passarelli di Napoli.

Measures of Noncompactness in Compact Operators and Differential Equations

M. Mursaleen
Department of Mathematics, Aligarh Muslim University
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Abstract

In this talk, we present a brief theory of measures of noncompactness and their applications in characterizing the compact matrix operators and in differential equations and integral equations. The classical measures of noncompactness are discussed and their properties are compared. The approaches for constructing measure of noncompactness in a general metric or linear space are described, along with the classical results for existence of fixed point for condensing operators. The most effective way in the characterization of compact operators between the Banach spaces is applying the Hausdorff measure of noncompactness. In this talk, we present some identities or estimates for the operator norms and the Hausdorff measures of noncompactness of certain operators given by infinite matrices that map an arbitrary BK-space into the sequence spaces c0, c, ℓ∞ and ℓ1 Many linear compact operators may be represented as matrix operators in sequence spaces or integral operators in function spaces [1]. Also several generalization of classical results are mentioned and their applications in various problems of analysis such as linear equation, differential equations, integral equations and common solutions of equations are discussed. Recently the measures of noncompactness are applied in solving infinite system of differential equations [3] and integral equations in sequence spaces [2].

References

1. J. Banas and M. Mursaleen, Sequence Spaces and Measures of Noncompactness with Applications to Differential and Integral Equations, Springer, 2014.
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Generalized Orlicz spaces and related PDE

Peter Alexander Hästö
University of Turku,

Abstract

Generalized Orlicz spaces include as special cases a wide range offunction spaces, such as Lebesgue space, Orlicz spaces, variable exponent spaces, double phase spaces and logarithmic perturbations of the aforementioned. Working in generalized Orlicz spaces involves some operations such as splicing the Orlicz functions that are not commonplace in the traditional Orlicz setting. In this talk, I explain some extensions to the Orlicz space theory which enable these operations and show that they may be useful even when there in the non-generalized Orlicz case, sometimes even yielding new results for classical Lebesgue spaces..

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3. P. Hästö: The maximal operator on Musielak–Orlicz spaces, J. Funct. Anal. 269 (2015), no. 12, 4038–4048.
4. P. Hästö and J. Ok: Maximal regularity for local minimizers of non-autonomous functionals , J. Eur. Math. Soc., to appear.
5. F.-Y. Maeda, Y. Mizuta, T. Ohno and T. Shimomura: Boundedness of maximal operators and Sobolev's inequality on Musielak–Orlicz–Morrey spaces. Bull. Sci. Math. 137 (2013), 76–96.

CONTRIBUTED TALKS

On the numerical solution of a semilinear Sobolev equation subject to nonlocal Dirichlet boundary condition

Abdeldjalil Chattouh[1] and Khaled Saoudi[2]
[1,2]ICOSI Laboratory, Department of Mathematics, Khenchela University, Algeria.
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Abstract

A semilinear Sobolev equation $\partial_t(u - \Delta u) - \Delta u = f(u)$ with a Dirichlet-type integral boundary condition $u|_{\partial\Omega} = \int Ku.dx$ is investigated in this contribution. Using the Rothe method which is based on a semi-discretization of the problem under consideration with respect to the time variable, we prove the existence and uniqueness of a weak solution. Moreover, a suitable approach for the numerical solution based on Legendre spectral-method is presented.

Keywords: Sobolev equation, Rothe method, time discretization, nonlocal boundary conditions, spectal method.

References:

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Coordination of Superconducting Magnetic Energy Storage and Superconducting Fault Current Limiter for Power Transmission System Transient Stability Enhancement

Abdelkrim Zebar[1] and Madani Lakhdar[2]
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Abstract

The use of a power transmission system near its operating limits can cause its instability when the fault occurs. The damping of the system's oscillations can be obtained by the classical means such as automatic voltage regulator and governor action but also by the resistive type superconducting fault current limiter (SFCL) and the superconducting magnetic Energy Storage (SMES). SFCL gives excellent technical performance when compared to conventional fault current limiters. The fast self-recovery from normal state to superconducting state immediately after the fault removal is an essential criterion for resistive type SFCL operation. Subsequently, the SMES damps the system's oscillations by exchanging the power with the system. Active power and/or reactive power can be consumed or supplied by the SMES according to the system requirement. In order to analyze the effect of the SFCL and SMES on the damping of the system's oscillations in more detail, several performance indices are considered. Simulation results show that both the SFCL and SMES can stabilize the interconnected power system during a severe fault. However, the control effect of the coordinated SFCL and SMES is much superior to that of the individual SFCL or SMES.

Keywords: Modelling of Superconducting fault current limiter (SFCL), Modelling of Superconducting magnetic energy Storage (SMES), Transient stability numerical analysis
and enhancement.

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12. A.Zebar and K. zehar, " Coordinated Control of SFCL and SVC for Power System Transient Stability Improvement » TELKOMNIKA Indonesian Journal of Electrical Engineering, Vol. 13, No. 3, pp. 431- 440, March 2015.

Literature review of energy consumption simulation software

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Abstract

The reduction of energy consumption and CO2 emissions is of a great importance. Total esidential energy consumption accounts for more than 40% of the total energy consumed by other sectors The residential sector is one of the biggest consumers of energy in every country, and therefore focusing on the reduction of energy consumption in this sector is very important. The energy consumption characteristics of the residential sector are very complicated and variables affecting the consumption are very wide and interconnected with each other, a more detailed models are needed to assess economics and techniques impacts of adopting energy efficiency and renewable energy technologies suitable for residential applications.

The aim of this paper is to review some of the modeling techniques used to model residential energy consumption. They are gathered in two categories: top-down and bottom-up. The top-down approach considers the residential sector as an energy sink and do not take in account the individual end-uses. It uses the previous aggregate energy values and regresses the energy consumption of the residential houses as a function of top-level variables such as macroeconomic indicators GDP (gross domestic product), unemployment, inflation, energy price, and general climate. The bottom-up approach uses the estimated energy consumption of a representative set of individual houses and extrapolated it to regional and national levels, and it can be summarized in two different methodologies: the statistical method and the engineering method.

Each of both techniques is based on different levels of data information, different calculation or simulation programs, and brings results with different applicability. Based on the strengths, shortcomings and purposes, an analytical review of each technique, is provided along with a review of models reported in the literature.

Keywords: Energy consumption, CO2 emmissions, modeling techniques, software by country, analysis of modeling sotware. Bottomup and top level models.

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Existence and Ulham stability of solutions for fractional integro-differential equation with integral boundary conditions

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Abstract

This paper deals with the stability results for solution of a fractional integro-differential problem with integral conditions. Using the Krasnoselskii's, Banach fixed point theorems, we proof the existence and uniqueness results. Based on the results obtained, conditions are provided that ensure the generalized Ulam stability of the original system. The results are illustrated by an example.

Keywords: Fractional integro-differential equation, existence, Ulham stability, nonlocal conditions, fixed point theorem, single valued maps.

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On probabilistic - fuzzy decision making criteria and their application for choose of financial operations.

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Abstract

It is known that when solving real problems, the income from financial markets is given with certain probabilities. The present paper presents the decision-making criteria of Bayes and Khoja Leman, whit different distribution, as well as the implementation of algorithms built using their R programming language.

Fuzzy analogues of these criteria have also been interpreted.

Keywords: decision making under uncitainity, probability, mathematical expectation, fuzzy sets.

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On Equiconvergence Rate for The One-Dimensional Dirac Operator

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Abstract

In this work, we consider one-dimensional irac operator

$$Du = Bu' + Q(x)u,\ \ u(x) = (u_1(x), u_2(x))^T,$$

on the interval $G = (0,2\pi)$, where $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, function from the class $Lr(G), r \in (0,\infty]$.

We study componentwise uniform eqiconvergence rate of spectral expansion for absolute conditinuous vectorfunction with respect to eigen-vector functions of Dirac operator D and trigonometric Fourier series expansion of this functions. Weestablish estimates for componentwise uniform equiconvergence rate on any compact $K \subset G = (0,2\pi)$.

Keywords: Dirac operator, equiconvergence, spectral expansion.

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Analysis of National Rice Stock Prediction in Meeting the Consumption Needs of Indonesian People Using Exponential Smoothing and Trend Moments

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Abstract

The Indonesia population has increased each year. There were 205.1 million people in 2008, and the number raised to 261.4 million people in 2017. This condition impresses on several commodities that must be provided by the Indonesian government so that the needs of the community are fulfilled sufficiently. The number of commodities affects the increasing amount of rice consumption. The total consumption of Indonesian rice reached 21,577,748 tons in 2008, and it became 22,847,706 tons in 2017.

The high level of rice consumption in the community resulted in an increase in rice demand. When the supply (stock) is not sufficiently in demand, there will be a scarcity, also the rise of price rice. Therefore, related institutions or the National Logistics Agency must be able to estimate rice consumption and procurement of rice needed for the coming year in Indonesia. Analysis of rice stock availability is deemed necessary to overcome the scarcity of rice stock availability in Indonesia or the National Logistics Agency later. We design an analysis to predict the rice stock availability needed for the coming year. We use the Exponential Smoothing and Trend Moment method to analyze the prediction for National Rice Stock to meet the Indonesian Consumption Needed.

The exponential smoothing method can predict the availability of rice for one coming year, where this calculation produces the lowest MAPE accuracy rate of 5,59% with an alpha smoothing parameter = 0,7, and the forecast result of national rice availability equals 46.230.452,49 tons. In using the trend moment method, the MAPE produced was 9,48% and resulted in forecasting the availability of national rice of 73.324.328,01 tons. A comparison of the two methods can be seen from the results of the calculation of the level of accuracy (MAPE) produced, and the exponential smoothing has the smallest MAPE results compared to the trend moment method. Prediction results with the lowest level of accuracy are expected to be considered for one year to come in overcoming the availability of national rice stocks even though there are factors that influence the ups and downs of the availability value.

Keywords: Prediction of rice availability, exponential smoothing analysis, trend moment analysis.

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On the solutions of the two preys and one predator type model approached by the Banach fixed point theory

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Abstract

The purpose of this paper is to discuss a special type of functional equation that describes the relationship between the predator animals and their two choices of prey with their corresponding probabilities. Our aim is to find the existence and uniqueness results of the proposed functional equation by using the Banach fixed point theorem. Finally, we give two illustrative examples to support our main results.

Keywords: Functional equations, predator-prey model, fixed points, Banach fixed point theorem.

Analyzing Attribute Control Charts for Defectives Based on Intuitionistic Fuzzy Sets

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Abstract

Control charts (CCs) are one of the most used statistical quality control techniques, which enable the controllers can measure the product specifications in the manufacturing process to determine the process is whether or not in the accepted limits (Feigenbaum, 1991). If the measured specifications are outside of the determined limits, CCs notice the system to take necessary precautions to keep the process at the desired level. The CCs can be classified based on the quality characteristics, which are measurable on numerical scales and not (Engin et al., 2008) and they can be classified into two groups based on data as "variable" or "attribute". Two well-known attribute control charts (ACCs) named *p* and *np* control charts are designed to measure the defectives during the manufacturing stages. If the process is deal with the number of defectives, then *np* control charts are used. Similarly, if the process deals with the defective rate, the *p* control charts are used.

In the traditional CCs, one of the most important issues is to represent the available data with the highest rate. Since the handled data may consist of uncertain information, ordinary *p* and *np* CCs have remained incapable of the ability to reflect the data. Moreover, the operators or the observers of the system can be hesitant while measuring these values during the data gathering process. Therefore, dealing with these problems can be realized by extending the ordinary CCs with useful tools. The fuzzy set theory (FST) is a tool, which enables to the representation of uncertainty by assigning membership function to an element by indicating the level of belongingness to a set (Zadeh, 1965). In the literature, classical fuzzy sets are used to extend *p* an *np c* control charts in many studies to increase the models' data representation and interpretation (Shu & Wu, 2010; Huang et al., 2012; Erginel, 2014; Sogandi et al., 2015). Since the FST are incapable of representing hesitancy, the models are not practical to include hesitancy during the calculations. Intuitionistic fuzzy sets (IFSs), which is an extension of FST is a way of representing not only the uncertainty in the data but also the hesitancy of the decision-makers (Atanassov, 1986). It is a useful tool to represent attribute information by using scales corresponded with intuitionistic fuzzy numbers (IFNs) during constructing the system structure. Comparing with the existed studies, the usage of IFSs enables the researchers to represent the hesitancy in their mathematical calculations. For this aim, two types of ACCs have been re-designed based on Ifs to improve their sensitiveness and flexibility.

In this study, an extension of *p* and *np* control charts with IFs are proposed and the design of these control charts based on IFs has been represented in detail. For this aim, control limits and center lines have been re-formulated by using the proposed extension. Moreover, a descriptive example is introduced to check the applicability of the proposed method.

For further studies, the other types of fuzzy extensions such as type-2, Pythagorean fuzzy sets, etc. can be used to design of *p* and *np* control charts can be analyzed.

Keywords: Fuzzy logic, intuitionistic fuzzy sets, *p* control chart, *np* control chart.

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Design of Control Charts for Number of Defects Based on Pythagorean Fuzzy Sets

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Abstract

Control charts (CCs) are completely useful techniques to monitor the process' stability and control. If the process is stable and in statistical control, it only displays a variation that is inherent to the process. So, usage of them is completely critical for the process's improvements. Two types of CCs are related to the total count of defects and they are called *u* and *c* control charts that are the designs of the non-measurable characteristics. The design of these CCs is based on the number of defects in a sample or a product. If the process is monitored based on the number of defects in a sample, *c* control charts are appropriate to use. In the same way, if the process monitored based on the number of defects per unit then *u* control charts can be used. If the process related to evaluations about defects includes some uncertainty, these types of attribute control charts are insufficient. So, they can be extended by using a tool to manage these uncertainties. The fuzzy set theory is a tool of representing uncertainty by assigning membership functions, which indicates the belonging an element to a fixed set (Zadeh, 1965). They are used to extend *u* and *c* control charts in many studies to increase the models' data representation and interpretation.

Pythagorean fuzzy sets (PFSs), which is an extension of intuitionistic fuzzy sets is a way of representing not only uncertainty in the data but also hesitancy of the decision-makers (Yager, 2013). It is a useful tool to represent attribute information by using scales corresponded with Pythagorean fuzzy numbers (PFNs). Comparing with the previous studies in the literature, the usage of PFs into CCs can improve the flexibility and sensitiveness of these charts.

For this aim, the design of control charts for the number of defects based on PFs has been analyzed in this paper. An extension of *u* and *c* control charts based on PFs are proposed and the design of these control charts has been detailed. For this aim, control limits and center lines have been re-formulated. Moreover, a descriptive example is introduced to check the applicability of the proposed method.

For further studies, the Pythagorean fuzzy u and c control charts can be analyzed to improve their control procedures by taking into account the control chart rules. For this aim not only the rule that checks any point outside of the control limits, but all of the special rules should also be analyzed based on PFs.

Keywords: Fuzzy logic, Pythagorean fuzzy sets, *u* control chart, *c* control chart.

Acknowledgment: This study is supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under Project Number 119K408.

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A Fifth Order of Accuracy Difference Scheme for Nonlocal Boundary Value Schrödinger Problem

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Abstract

In this study, nonlocal boundary value Schrödinger type problem in a Hilbert space with the self-adjoint operator is investigated. Single step fifth order of accuracy difference scheme for the numerical solution of this problem is presented. The main theorem on the stability of this difference scheme is established. In applications, theorems on the stability of difference schemes for several nonlocal boundary value problems are presented. Numerical results are given.

Keywords: Difference scheme, stability, Schrödinger problem

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On approximation of signals in the generalized Zygmund class using product mean of conjugate derived Fourier series

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Abstract:

Signal analysis describes the field of study whose objective is to collect, understand and deduce information and intelligence from various signals. Now-a-days, the analysis of signals is a fundamental problem for many engineers and scientists. In the recent past, we have seen the applications of mathematical methods such as Probability theory, Mathematical statistics etc. in the analysis of signals. Very recently, approximation theory has got a large popularity as it has given a new dimension in approximating the signals. The estimation of error functions in Lipschitz and Zygmund space using different summability techniques of Fourier series and conjugate Fourier series have been of great interest among the researchers in the last decades.

In the present article, we have established a result on degree of approximation of function in the generalized Zygmund class $Z_l^{(m)}$, $(l \geq 1)$ using $(E, r)(N, q_n)$ −mean of conjugate derived Fourier series.

Keywords: Degree of approximation, Generalized Zygmund class, Fourier series, Conjugate Fourier series, Conjugate Derived Fourier series, (E, r)- Summability mean, (N, q_n) – Summability mean, $(E, r)(N, q_n)$-Summability mean.

Lupa\c{s} type Bernstein operators on triangles based on quantum analogue

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Abstract

The purpose of the paper is to introduce a new analogue of Lupa\c{s} type Bernstein operators $\big(B^{u}_{m,q}f\big)(u,v)$ and $\big(B^{v}_{n,q}f\big)(u,v)$, their products $\big(P_{mn,q}f\big)(u,v)$ and $\big(Q_{nm,q}f\big)(u,v)$ and their Boolean sums $\big(S_{mn,q}f\big)(u,v)$ and $\big(T_{nm,q}f)(u,v) $ on triangle T_{h}, which interpolate a given function on the some edges and at the vertices of triangle using quantum analogue. We deal with convergence properties of the sequences of all above operators. It is proved that $\big(B^{u}_{m,q_{m}}f\big)(u,v)$ converges uniformly to $f(u,v)$ for any $f(u,v)\in C(T_{h})$ if and only if $q_{m}\rightarrow 1$. Based on Peano's theorem and using modulus of continuity, the remainders of the approximation formula of corresponding operators are evaluated. It has been shown that parameter q will provide more flexibility for approximation.

Keywords: Lupa\c{s} q-Bernstein operators; product operators; Boolean sum operators; modulus of continuity; Peano's theorem; error estimation.

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Intuitive Approximation for Renewal Reward Process with $\Gamma(g)$ Distributed Demand

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Abstract

This study is motivated by an interest in observation of some major characteristics of a semi-Markovian stochastic model of type (s,S) when demand random variables are in a special class known as the class of Γ (g). By using the approximation results for renewal function providing by Mitov and Omey [2], we obtained intuitive approximation for ergodic distribution of a stochastic process representing a classical semi-Markovian inventory model of type (s,S). Considered model has been investigated with logistic distributed random variables previously [1]. We investigate the current problem here with the class of Γ (g) distributed random variables rather than a single distribution like logistic.

Keywords: Γ (g) class of distributions, Heavy tailed distributions, semi-Markovian inventory model, Intuitive approximation.

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Classification of land cover by spectral and textural characteristics

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Abstract

The rapid development of computer science and applications of the broad spectrum of programmable systems (eg, Matlab, Matmatics, Mapple, etc.) have enabled the use of satellite data in different identity problems and recognition. The application of new approaches and a comparative analysis of the results obtained with known results is of great importance, both from a theoretical and applied point of view. This article is devoted to the classification of soil plants according to spectral characteristics obtained by remote sensing methods. The problem of preliminary analysis of the informativeness of the spectral features themselves and the textural features calculated on their basis is considered. The LBP (Local binary pattern) histogram method is used to calculate the values of textural attributes. The LBP method describes the spatial structure of the image according to the local structure of the texture. In this article, the cotton field of the Hajigabul region of the Azerbaijan Republic was taken as the study area. The classification of land cover was carried out using metric distances and classification methods. The data obtained on the spectral channels (blue, green, red, and near infrared) were used as spectral data using an unmanned aerial vehicle XA-Rotor-1000 X-8. The "pdist", "linkage" and "cluster" functions included in the MATLAB batch programs were used to perform the calculations that led to the definition of which class each object belongs toIn 11 the calculations, the Euclidean distance was used as the metric distance. The "nearest neighbor" classification algorithm was applied. Given comparative mathematical approaches to the solution of the classification of plants-soil using remote data. Investigated the possibility of applying the automatic classification of soil plants according to the remote data of the classification and recognition methods included in the Matlab software system. It is shown that in the problems of classification of objects, "The statement that the objective decision rule, when a fragment is taken as a whole, more efficient than the pixel decision rule," is not always true.

Keywords: remote data, classification, metric distance, soil type, identification, recognition, textural signs.

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One Class of Linear Fredholm Operator Equations of the Third Kind

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Abstract

In this paper, we are applying a new approach to prove that the solution of the linear Fredholm operator equation of the third kind is equivalent to the solution of the linear Fredholm operator equation of the second kind with a certain conditions.

Keywords: Linear Fredholm operator equation, integral equation, third kind, second kind, solution, Banach space.

In work we consider the linear operator equation of the third kind

$$P(t)u(x) = \lambda\int_a^b K(x,y)u(y)dy + f(x), \quad x \in [a,b], \tag{1}$$

where $P(x)$ is given continuous function on $[a,b]$, for all $(x, y)\in G=[a,b]\times[a,b]$ the operator $K(x,y)\in L(X)$, X is a Banach space, $L(X)$ is the space of linear bounded operators acting from X into X, $C([a,b];X)$ is the Banach space of continuous functions defined in $[a,b]$ attaining the values in X with the norm $\|u(t)\|_C = \sup_{t\in[a,b]}\|u(t)\|_X$, $f(x)$ is given continuous function from $C([a,b];X)$, $u(x)$ is sought continuous function on $[a, b]$, λ is a real parameter, $P(x_i) = 0$, $x_i\in[a,b]$, $i = 1, 2, ..., m$. The integral is taken in the Bochner sense [4]. It is proved that the solution of the operator equation (1) in $C([a,b];X)$ is equivalent to the solution of the linear operator equations of the second kind under certain conditions.

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A Different Approach to Statistical Manifolds

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Abstract

The purpose of this study is to create a new statistical manifold with the help of the anti-Kahler manifold. We analyze some properties of the curvature tensor field and give an example for this new manifold.

Keywords: Statistical manifolds , α-conections, anti-Kahler manifold.

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On frame properties of iterates of a multiplication operator

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Abstract

Dynamical sampling that is a relatively new research topic in applied harmonic analysis has attracted considerable attention in recent years. One of the central problems in dynamical sampling is investigation of frame properties for families of elements obtained by iterates of operators.

Note that investigation of basicity properties (completeness, Schauder basicity, frameness, etc.) of iterates of operators is problematic even in the case of well known "standard" operators.

This note is dedicated to the study of frame properties of iterates of a multiplication operator. The main goal of this paper is to show that the orbit $\left\{T_{\varphi}^{n} f\right\}_{n=0}^{\infty}$ of the multiplication operator $T_{\varphi} f(t) = \varphi(t) \cdot f(t),\ f \in L_2(a,b)$ cannot form a frame for the space $L_2(a,b)$ for any measurable generator $\varphi(t)$ and any $f \in L_2(a,b)$.

The main result of this note is the following theorem.

Theorem. *Let* $\varphi(t)$ *be any measurable function and* $f(t)$ *any square summable function on* (a,b). *The system* $\left\{T_{\varphi}^{n} f\right\}_{n=0}^{\infty}$ *cannot be a frame in* $L_2(a,b)$.

Acknowledgement. The authors are grateful to Professor B.T. Bilalov for encouraging discussion.

Keywords: Dynamical sampling, operator orbit, frame, Schauder bases, system of powers, Lebesgue spaces.

On the completeness and minimality of the exponential system with degenerate coefficients

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Abstract

We consider completeness and minimality in $L_p(-\pi,\pi), 1<p<\infty$ of systems of the form $\left\{\omega(t)e^{int}\right\}_{n\in Z}$, where $\omega(t)=|t-t_1|^{\alpha_1}|t-t_2|^{\alpha_2}\prod_{j=3}^{r}|t-t_j|^{\alpha_j}$, $t_j\in[-\pi,\pi]$ for all $1\le j\le r$ and $\alpha_1,\alpha_2\in\left[\frac{1}{q},1+\frac{1}{q}\right)$, $\alpha_j\in\left(-\frac{1}{p},\frac{1}{q}\right)$ for all $3\le j\le r$. Note that, in the sequel, we denote by Q the set of all rational numbers.

Theorem. *The following statements hold:*

1) *If* $\frac{t_2-t_1}{\pi}\notin Q$, *then the system* $\left\{\omega(t)\cdot e^{int}\right\}_{n\in Z/\{k_1;k_2\}}$ *is complete and minimal for any choice of indices* k_1 *and* k_2;
2) *If* $|t_2-t_1|=2\pi$, *then the system* $\left\{\omega(t)\cdot e^{int}\right\}_{n\in Z/\{k_0\}}$ *is complete and minimal for any integer* k_0;
3) *If* $t_2-t_1=2\pi\frac{k}{m}$, *where* $m\neq 1$ *and* $(k,m)=1$, *then* $\left\{\omega(t)\cdot e^{int}\right\}_{n\in Z/\{k_1;k_2\}}$ *is complete and minimal if and only if* $k_2\neq k_1(\text{mod}\, m)$.

The authors are grateful to Professor B.T.Bilalov for encouraging discussion.
Keywords: completeness, minimality, exponential system with degenerate coefficients.

On the existence of a integral solution of the inverse problem for equation of parabolic type

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Abstract

For an equation of parabolic type, we consider the inverse problem, which reduces to the following system of integral equations:

$$u(x,t)=\varphi(x)+\int_0^t\int_D \Gamma(x,t;\xi,\tau)\left[f(\xi,\tau,u)+\Delta\varphi(\xi)-\qquad -c(\xi)H(\xi,t)\right]d\xi d\tau+\int_0^t\int_{\partial D}\Gamma(x,t;\xi,\tau)\rho(\xi,\tau)d\xi d\tau,$$

$$c(x)=\left[\varphi(x)+\Delta h(x)+\int_0^T f(x,t,u)dt-u(x,T)\right](h(x))^{-1},$$

$$(x,t)\subset\Omega=D\times(0,T],0<T=const,\ x\in\overline{D}=D\cup\partial D,\ D\subset R^n.$$

Here $\Gamma(x,t;\xi,\tau)$ is a fundamental solution to the equation $u-\Delta u=0$.

By given functions $f(x,t,u),\varphi(x),\psi(x,t,u),h(x)$ it is required to determine a pair of functions $\{c(x),u(x,t)\}$ which is an integral solution of the inverse problem and a solution to the system of integral equations, where $c(x)\in C(\overline{D})$, $u(x,t)\in C^{2,1}(\Omega)\cap C^{1,0}(\bar{\Omega})$.

The existence of a solution to the system of integral equations is carried out by the method of successive approximations.

Keywords: Equation of parabolic type, inverse problem, system of integral equations, integral solution, method of successive approximations.

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Investigating the Stock Control Model of Type (s, S) with Dependent Components

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Abstract

In this study, a semi-Markov stock control model of type (s, S) with dependent components is mathematically constructed and investigated. The classical stock control model assumes that the inter-arrival times $\{(\xi_n), n \geq 1\}$ and the demands $\{(\eta_n), n \geq 1\}$ are mutually independent random variables. However, stochastic model of type (s, S) which expresses some real-world problems should be investigated using the stochastic processes with dependent components. The mathematical structure for this type of processes is very complex. As a result, studying this model under the dependence assumption is very difficult. To partially eliminate this difficulty, a stochastic process (X(t)) describing the model of type (s, S) with dependent components is constructed and stationary distribution of the process is studied. The exact expression for the ergodic distribution $(\hat{Q}_X(x))$ of the process X(t) is obtained as follows:

$$\hat{Q}_X(x) = 1 - \frac{U(S-x)}{U(S)} + \frac{1}{aU(S)}\{G(S) * U(S) - G(S-x) * U(S-x)\}$$

Here $U(x) \equiv \sum_{n=0}^{\infty} F^{*n}(x)$; $F^{*n}(x)$ is a convolution product of $F(x)$; $F(x) \equiv P\{\eta_1 \leq x\}$; $a \equiv E(\xi_1)$;
$G(t) * U(t) = \int_{z=0}^{t} G(t-z)\, dU(z)$; $G(t) \equiv \int_{v=0}^{t} K(v)\, dF(v)$; $K(v) \equiv E(\xi_1/\eta_1 = v) - a$.
The dependence between inter-arrival times and the amount of demands is expressed by the means of the function $K(v)$. Moreover, the asymptotic expansion for the ergodic distribution of the process X(t) is obtained under the assumption that $K(v)$ is a linear function of v. In addition, the weak convergence theorem for the ergodic distribution of the process is proved. Also, the exact expressions and three-term asymptotic expansions are found for all the moments of the stationary distribution of the process X(t).

Keywords: Stock control model of type (s, S); Dependent components; Ergodic distribution; Weak convergence; Asymptotic Expansion.

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The Proper Class Generated Projectively by G-Semiartinian Modules

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Abstract

In this presentation, we introduced and studied a new type of proper class. A module M is called g-semiartinian if every non-zero homomorphic image of M contains a simple singular module. A submodule N of a module M is called gd-closed if there is S⊂M such that S∩N = 0 and M/(S⊕N) has projective socle. The exact sequence $\boldsymbol{E}: \mathbf{0} \to \boldsymbol{A} \to^{f} \boldsymbol{B} \to \boldsymbol{C} \to \mathbf{0}$ is called gd-closed if Im f is gd-closed in B. The class of all gd-closed exact sequences is denoted by GD -Closed. The class GD -Closed forms a proper class in the sense of [3]. A module M is GDC-flat if every exact sequence ending with M is gd-closed. First of all, it is obvious that projective modules are GDC-flat. Furthermore, nonsingular modules and modules with projective socle are less obvious examples of GDC-flat modules. We show that the class GD-Closed is generated projectively by g-semiartinian modules. We study right PS rings by GDC-flat modules. We show that R is right PS ring if and only if every submodule of an GDC-flat module is GDC-flat if and only if the subprojectivity domain $\underline{\boldsymbol{Br^{-1}}}$(Y) is closed under submodules for each g-semiartinian right module Y if and only if every right module has an epic GDC-flat envelope if and only if every g-semiartinian right module has an epic projective envelope.

MSC 2010: 16D10, 16D40
Keywords: G-semiartinian modules, Subprojectivity domain, PS ring
Acknowledgement:
This work was supported by Research Fund of the Cukurova University. Project Number: 12308

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On Inclusion Relations GD-Closed And Some Other Proper Classes

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Abstract

A submodule N of a module M is called gd-closed if there is a submodule S⊂ M such that S ∩N =0 and the socle of M/(S⊕ N) is projective. Gd-closed submodules determine a proper class of short exact sequences and we denote this proper class by **GD-Closed**. In this paper, we study the inclusion relations between this proper class and some other proper classes.

Keywords: GD-Closed, Proper class, g-semiartinian modules
Acknowledgement:
This work was supported by Research Fund of the Cukurova University. Project Number: 12308

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On the Z-Symmetric Manifold with Conharmonic Curvature Tensor in Special Conditions

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Abstract

The object of the present paper is to study the Z-symmetric manifold with conharmonic curvature tensor in special conditions. In this paper, we prove some theorems about these manifolds by using the properties of the Z-tensor.
Keywords: Conharmonic curvature tensor, Z-symmetric tensor, Codazzi tensor, torse-forming vector field, recurrent tensor.

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FINITE ELEMENT SOLUTIONS OF THE BURGERS EQUATION

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Abstract

In this study, a central difference method in time and the Galerkin finite element method in space are applied to solve the Burgers equation. The resulting system of the nonlinear equations obtained at each time step is solved by using programming codes in MATLAB. In order to show the efficiency of the presented method, the numerical solutions obtained for various values of viscosity at different times are compared with the exact solutions.

Keywords: Burgers equation, the central difference method, Galerkin finite element method

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Linear Codes over the Ring $\mathbb{Z}_8 + u\mathbb{Z}_8 + v\mathbb{Z}_8$

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Abstract

In this work, we introduce the ring $R = \mathbb{Z}_8 + u\mathbb{Z}_8 + v\mathbb{Z}_8$, where $u^2 = 0$, $v^2 = 0$, $uv = vu = 0$ (the ring R can be viewed as the quotient ring $\mathbb{Z}_8[u,v]/\langle u^2 - u, v^2 - v, uv - vu \rangle$) over which the linear codes are studied. We also defined the Lee weight and Lee distance of an element of R and investigate the generator matrices of the linear code and its dual.

Keywords: Linear codes over rings, Lee weight, generator matrix, duality.

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Determining the Best Prices for Two Substitues using Interval Valued Triangular Fuzzy Numbers

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Abstract

Suppose that a firm produces two substitutes, namely good-1 and good-2. The demand functions that link the quantities demanded (x_1 and x_2) and the prices (P_1 and P_2) are given by $x_1 = a_1 - a_2P_1 + a_3P_2$ (where $a_1 > 0$ and $0 \leq P_1 \leq a_1/a_2$) and $x_2 = b_1 + b_2P_1 - b_3P_2$ (where $b_1 > 0$ and $0 \leq P_2 \leq b_1/b_2$). The coefficients are fuzzificated using interval valued triangular fuzzy numbers, then the best prices and the optimal revenue are calculated for the firm.

Keywords: Microeconomics, Fuzzy demand, Fuzzy total revenue, Interval valued fuzzy numbers

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Consumer Surplus and Producer Surplus of the Linear Demand and Supply Functions using Interval Valued Triangular Fuzzy Numbers

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Abstract

Let the linear demand function be $p = a - bx$, $0 \leq x \leq \frac{a}{b}$ and the supply function be $p = e + gx$, $x \geq 0$ where a, b, e, g are positive constants and $e < a$. In this talk, we fuzzified the the quantity x by using the interval valued triangular fuzzy numbers. After making the calculations we apply the signed distance defuzzification method to get the crisp results.

Keywords: Consumer surplus and producer surplus; Interval valued triangular fuzzy numbers; Signed distance defuzzification method.

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COFINITELY WEAK e-SUPPLEMENTED MODULES

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Abstract

In this work, R will denote an associative ring with unity and all module are unital left R modules. Let M be an R module. If every cofinite essential submodule of M has a weak supplement in M, then M is called a cofinitely weak e-supplemented (or briefly cwe-supplemented) module. In this work, some properties of these modules are investigated.
Key words: Co…nite Submodules, Essential Submodules, Small Submodules, Supplemented Modules.

2010 Mathematics Subject Classification: 16D10, 16D70.

Results

Proposition 1 Every cofinitely essential supplemented module is cwe-supplemented.
Proposition 2 Every essential supplemented module is cwe-supplemented.
Proposition 3 Every weakly essential supplemented module is cwe-supplemented.
Proposition 4 Every finitely generated cwe-supplemented module is weakly essential supplemented.
Proposition 5 Every cofinitely weak supplemented module is cwe-supplemented.
Proposition 6 Every weakly supplemented module is cwe-supplemented.
Proposition 7 Every cofinitely supplemented module is cwe-supplemented.
Proposition 8 Every supplemented module is cwe-supplemented.
Proposition 9 Let M be a cwe-supplemented module. If every nonzero submodule of M is essential in M, then M is cofinitely weak supplemented.

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Stability Analysis of a Linear Neutral Differential Equation

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Abstract

In this study, a special case for linear neutral differential equation with a constant delay is considered. By using the characteristic equation the stability analysis is made. The results are obtained by Routh-Hurwitz criterion and the Sturm sequence.

Keywords: Linear neutral differential equation, stability analysis, characteristic equation, Routh-Hurwitz criterion, Sturm sequence.

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Development of Multiple Linear Regression Forecasting Approach for The Number of Customers in A High Speed Train Line.

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Abstract

Correct and reliable order forecast has an important role in firms' elevation in efficiency. As in every field also in Food industry the order forecasts are very important. Especially in one day shelf life products it's a must to make accurate order forecast and prevent food waste excess.In this study an approached was made for the forecast passenger numbers which in the case the meal order for the meal provider firm for high speed trains.

The estimation with multiple regression of the passenger muchness who traveled Ankara-İstanbul and İstanbul-Ankara legs in 'Business Plus' and 'Economy Plus'classes are mantioned according to 2016-2019 year basis datas of a subcontactor company of The Republic of Turkey State Railways. The accurate Passenger number estimation will create a better meal order and will become a financial income. Many Estimation models are practiced according different ways to apply data classifications and choices of regression model variants. Very accurate passenger number estimation model alternatives are developed as a result of the avarege miscalculation data of the best three practiced models.

Keywords: Demand Forecast, Multiple Linear Regression Method ,Passenger numbers forecast

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On the existence and uniqueness of solutions for fractional integro-differential equation with nonlocal condition

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Abstract

In the present paper we study a Cauchy problem for Caputo's fractional integro-differential equation with nonlocal condition in Banach Space. We shall prove existence and uniqueness results by using Banach fixed point and Krasnoselskii's fixed point theorems. Some illustrative examples are presented for the justification of our main results.

Keywords: Nonlocal condition, Banach space, fixed point theorem, fractional integro-differential equation.

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The Spacelike Bonnet Surfaces in Lorentzian 3-Space

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Abstract

In this paper, we generalize the criteria which were given by Z.Soyuçok in the 3-dimensional Euclidean Space to the Lorentzian 3-Space for the Spacelike surfaces. By the aid of this criteria, we investigate Spacelike Bonnet Surfaces and their associate surfaces. Finally, we consider the classification of helicoids which were given by Beneki et al. We give these as examples to illustrate our results. They satisfy our conditions as Spacelike Bonnet Surfaces. Also, we determine their associate surfaces.

Keywords: Bonnet Surface, Spacelike surface, Helicoid, Associate surface, A-net.

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Mode Matching Technique for Analysis of Sound Wave in an Infinite Duct with Different Linings

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Abstract

Propagation of sound waves by an infinite circular cylindrical duct with different absorbing linings is investigated by using the Mode Matching Technique. An analytical solution for the field terms are determined in form of eigenmodes which are matched across the boundary of each junction discontinuity. Graphical results are obtained to show the effect of the waveguide radius and acoustic absorbing lining on the propagation phenomenon. Also, the reflection coefficient is compared with a study existing in the literature and perfect agreement is observed.

Keywords: Mode matching technique, sound propagation, absorbing lining, duct.

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eg-RADICAL SUPPLEMENTED MODULES

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Abstract

In this work, R will denote an associative ring with unity and all module are unital left R modules. Let M be an R module. If every essential submodule of M has a g-radical supplement in M, then M is called an essential g-radical supplemented (or brie‡y eg-radical supplemented) module. In this work, some properties of these modules are investigated.

Key words: Essential Submodules, g-Small Submodules, Generalized Radical, g-Supplemented Modules.

2010 Mathematics Subject Classi…cation: 16D10, 16D70.

Results

Proposition 1 Every g-radical supplemented module is eg-radical supplemented.
Proposition 2 Every g-supplemented module is eg-radical supplemented.
Proposition 3 Every essential g-supplemented module is eg-radical supplemented.
Proposition 4 Let M be an eg-radical supplemented module. If every nonzero submodule of M is essential in M, then M is g-radical supplemented.
Proposition 5 Every essential supplemented module is eg-radical supplemented.
Corollary 6 Let M = M1+ M2+ ::: + Mn. If Mi is essential supplemented for every i = 1; 2; :::; n, then M is eg-radical supplemented.
Corollary 7 Let M be an essential supplemented module. Then every …nitely M generated module is eg-radical supplemented.
Corollary 8 Let R be a ring. If RR is essential supplemented, then every …nitely generated R module is eg-radical supplemented.

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r-SMALL SUBMODULES

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Abstract

In this work, every ring have unity and every module is unital left module. Let M be an R module and N M. If N RadM, then N is called a radical small (or brie‡y r-small) submodule of M and denoted by N r M. In this work, some properties of these submodules are given.

Key words: Small Submodules, Maximal Submodules, Radical, Supplemented Modules.

2010 Mathematics Subject Classi…cation: 16D10, 16D70.

Results

Proposition 1 Let M be an R module and N M. If N r M, then N M.
Proposition 2 Let N M. If N M and RadM is a supplement submodule in M, then N r M.
Proposition 3 If N r M, then N K for every maximal submodule K of M.
Proposition 4 Let M be an R module and N K M. If N r K, then N r M.
Proposition 5 Let M be an R module and N K M. If K r M, then N r M.
Proposition 6 Let M be an R module and N; K M. If N r M, then (N + K) =K r M=K.
Proposition 7 Let f : M !N be an R module homomorphism. If K r M, then f (K) r N.
Lemma 8 Let M be an R module and K; L M. If N r K and T r L, then N + T r K + L.
Corollary 9 Let M1; M2; :::; Mk M. If N1 r M1, N2 r M2, ..., Nk r Mk, then N1 + N2 + ::: + Nk r M1 + M2 + ::: + Mk.

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Korovkin-type theorems and their statistical versions in the weighted grand-Lebesgue spaces

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Abstract

In the paper we study Korovkin-type theorems and their statistical versions in the weighted in grand-Lebesgue spaces $L_{p),\rho}$. Based on shift operator, we define the subspace $G_{p),\rho}(0,1)$ of the space. $L_{p),\rho}(0,1)$, where continuous functions are dense, and study some properties of the functions belonging to this space *The analogs of Korovkin theorems are proved in.* $1 < p < +\infty$, when weight function ρ satisfy the Muckenhoupt condition.

Keywords: weighted grand-Lebesgue space, Muckenhoupt condition Korovkin theorems, statistical convergence

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On estimates of the fundamental solution of the degenerate parabolic equations

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Abstract

In the literature, the termin of local properties is used usually to refer the Holder regularity, Harnack's inequality, two side estimates of fundamental solution and etc. results for 2-nd order parabolic equations, also for the cases of its degenerate and quasilinear analogues (see, e.g. [1], [2]). This abstract relates to the equation

$$\frac{\partial}{\partial x_j}\left(a_{ij}(t,x)\frac{\partial u}{\partial x_i}\right)-\frac{\partial u}{\partial t}=0 \qquad (1)$$

with the uniform degeneratacy condition

$$\frac{1}{C}\omega(t,x)|\xi|^2 a_{ij}(t,x)\xi_i\xi_j \le C\omega(t,x)|\xi|^2 \qquad (2)$$

for $C>1$, $\forall\xi\in\Re^n,(t,x)\in D$, and D be a bounded domain in half-space $\{t<t_0\}$.

Concerning the function $\omega(t,x)$ to be a measurable positive function and some Muckenhoupt's condition all over special cylinders and some additional assumptions are assumed in order to get the following results. Under the mentioned conditions on the degeneration $\omega(t,x)$ the estimates from below and upper have been stated for the fundamental solution of (1).

Keywords: regularity of solutions, Holder regularity, Harnack's inequality, fundamental solutions, weak solutions, qualitative properties of solutions, a prior estimates.

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IGAMSA: An Improved Genetic Algorithm Applied to Multiple Sequence Alignment Problem

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Abstract

Multiple sequence alignment (MSA) is a pre-processing tool in the subsequent analyses of protein families. It is has been identified as one of the challenging tasks in bioinformatics. It allows comparison of the structural relationships between sequences by simultaneously aligning multiple sequences and constructing connections between the elements in different sequences. The main problem in MSA is its exponential complexity with the considered input data set. Finding the optimal alignment of a set of sequences is known as a NP-complete problem. It classified as a combinatorial optimization problem, which is solved by using computer algorithms. These algorithms lead to represent, to process, and to compare genetic information to determine evolutionary relationships among living beings. At present, the iterative and stochastic algorithms have been increasingly used to solve the MSA problem. These approaches can improve the multiple sequence alignment through a series of iterations until the solution doesn't become better any longer. In this study, we aim to present an Improved Genetic Algorithm called (IGAMSA) to find an approximate solution to the multiple sequence alignment problem using some BAliBASE benchmarks.

Keywords: Multiple sequence alignment, NP-complete problem, Iterative and stochastic algorithms, IGAMSA, BAliBASE benchmarks.

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Binary Cluster Analysis For Real Data

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Abstract

Since binary clustering method is a method that allows clustering rows and columns at the same time in data analysis, it has been widely used in recent years. In this study, after the binary clustering method and the algorithms used for this method are briefly explained, an application is made for a real data set and the results are evaluated.

Keywords: Binary Clustering, Bimaks Algorithm, CC Algorithm.

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A Finite Difference Method to Solve a Special Type of Second Order Differential Equations

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Abstract

In this study, we give a finite difference sheme to solve a special type of second order differential equations. Our numerical method based on finite difference relation which is obtained the Lagrange polynomial interpolations.

Keywords: Finite difference method, boundary value problem, Lagrange polynomial interpolation

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Exact solutions and conservation laws of weakly dissipative modified two-component Dullin-Gottwald-Holm system

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Abstract

The Dullin-Gottwald-Holm equation models the unidirectional propagation of shallow water waves over a flat bottom. The generalised weakly dissipative modified two-component Dullin-Gottwald-Holm system is analyzed by using the Lie symmetry approach. The exact solutions of weakly dissipative modified two-component Dullin-Gottwald-Holm system are obtained in the form of power series and trigonometric functions. The conservation laws are obtained with the help of multiplier approach. The 3D representations of obtained solutions are also shown.

Keywords: weakly dissipative, two-component Dullin-Gottwald-Holm system, Lie symmetry approach, exact solutions, conservation laws.

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A Fourier Pseudospectral Method for the Improved Boussinesq Equation with Second-Order Accuracy

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Abstract

In this talk, a Fourier pseudospectral method is proposed for solving initial and boundary value problem for the Improved Boussinesq equation. The numerical scheme is based on a second-order finite difference in time and a Fourier pseudospectral method in space. Extensive numerical experiments such as propagation of a single solitary wave, the interaction of two solitary waves and single wave splitting are reported to demonstrate rich dynamics of the improved Boussinesq equation.

Keywords: Improved Boussinesq equation, Fourier pseudospectral method, Single solitary wave, Interaction of solitary waves, Single wave splitting.

A New Laplace-Type Integral Transform and Its Applications

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Abstract

In the present paper, the authors introduce a new integral transform,

$$\mathcal{L}_{\alpha,\mu}\{f(t);y\} = \int_0^\infty t^{\alpha-1}e^{y^\mu t^\mu} f(t)dt \quad (Re\alpha > 0, Re\mu > 0)$$

and consider its special case for $\alpha = \mu$, which is defined as

$$\mathcal{L}_{\mu}\{f(t);y\} = \int_0^\infty t^{\mu-1}e^{y^\mu t^\mu} f(t)dt \quad (Re\mu > 0).$$

Several simple theorems and results that are dealing with general properties of the $\mathcal{L}_{\alpha,\mu}$- and $\mathcal{L}_{\mu}$-integral transforms are proved. The existence theorem, convolution theorem, inversion theorem and Parseval type theorems for the $\mathcal{L}_{\alpha,\mu}$- and $\mathcal{L}_{\mu}$-integral transforms are given. These transforms are used for solution of a differential and an integral equation. Illustrative examples are also given.

Keywords: Laplace Transform, Parseval-Goldstein Type Theorems, Convolution, Differential Equation, Integral Equation.

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About One Property Of The Numerical Range of Two-Parametric Spectral Problem

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Abstract

The study of the structure of the numerical range in the multiparameter problem plays an important role in the study of the spectrum by the variational method [1]. In this paper it is proved that the numerical range is closed under the condition of left definiteness.

Keywords: multiparameter eigenvalue problems, spectr, numerical range, definiteness conditions..

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On the Hankel Transform Using HOL-Light Theorem Prover

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Abstract

Hankel transform is an integral transformation which includes Bessel functions as the kernel when solving axisymmetric problems in the cylindrical polar coordinates. It is applicable in wide variety of science and engineering areas such as optical data processing, generation of diffusion profiles, boundary value problems. In this study, we plan to formalize Hankel transform and its useful properties by using the HOL-Light theorem prover. HOL-Light has a library of transform methods as well as a reach set of mathematical theories such as differential, integration, transcendental, topology, complex numbers, Lp spaces and vector theories of multivariable calculus.

Keywords: Hankel transform, Higher-order logic, HOL-Light

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Design of EWMA and CUSUM Control Charts Based On Type-2 Fuzzy Sets

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Abstract

With the increase in the variety of products and services in the market, quality has become one of the leading factors. For this reason, comprehending and improving quality play an important role in attracting customers and staying competitive in the market. Since adopting quality as an essential part of business strategies brings significant benefits, many approaches have been developed to improve the quality of products and services. A Shewhart control chart (SCC), one of the most preferred techniques in statistical process control, is utilized to determine whether there are unusual sources of variability in a process or not. The systematic use of this technique is quite efficient to reduce such variations, as it will enable corrective measures to be taken to eliminate these unusual sources of variability (Montgomery, 2012; Burr, 1976; Leavenworth & Grant, 2000). It is a completely critical tool to monitor process' stability. However, the use of SCC has one drawback, which is that it disregards the information provided by the sequence and utilizes only the information given in the final sample observation. It consequently causes SCCs to be relatively insensitive to small process shifts (Montgomery, 2012; Yang et al., 2011). For this reason, Cumulative Sum (CUSUM) (Page, 1954), and Exponentially Weighted Moving Average (EWMA) control charts (Roberts, 1959) have been proposed to overcome this drawback and to effectively handle such process shifts.

The effectiveness of these control charts (CCs) depends on the accuracy of the available data. However, in most of the real-world problems, there are uncertainties in the processes related to measurement systems or operators. The fuzzy set theory (FST), proposed by Zadeh (1965), have been commonly employed in many fields to deal with vague and imprecise information. FST, handling uncertainty by defining membership degrees, have been integrated to control charts in the literature. Although this tool can manage the process related to uncertainty, the extensions of FST can be used to improve their ability. For this aim, one of FST extensions named type-2 fuzzy sets has been used to design of control charts. The main advantage of type-2 fuzzy sets, proposed by Zadeh (1975), is that they can effectively model uncertainty even when the membership functions are not crisp (Kilic and Kaya, 2015). Therefore, in this study, type-2 fuzzy sets are integrated into EWMA and CUSUM charts to design these control charts to be able to handle the uncertainties in the processes and measurements. For this aim, the control limits and center lines have been re-formulated based on type-2 fuzzy sets. Furthermore, an illustrative example is provided to show the applicability of the proposed techniques.

For future research directions, defuzzification procedure of type-2 fuzzy sets can be analyzed to use them into CCs. For this aim, the design of control charts can be considered with respect to rules of control charts that check whether the process is stable.

Keywords: Quality control, Type-2 fuzzy sets, EWMA, CUSUM.

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Boundary Perturbations in Laplace and Steklov Eigenproblems

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Abstract

In this study, we have considered the Laplace and Steklov eigenvalue problems involving the Laplace operator that is of fundamental importance in many physical phenomena. We analyze both analytically and numerically the effect of boundary perturbations on the eigensolutions of these eigenvalue problems. The convergence between the perturbed solution and the original solution is investigated according to the perturbation parameter by introducing a set of differing perturbations to various one and two-dimensional domains. Taylor's series expansions of the errors between the two solutions are used to obtain this convergence behavior. The theoretical results are confirmed by numerical experiments and are compatible with the results obtained for the corresponding boundary value (source) problem in [1]. Besides, we investigate numerically the convergence properties of the finite element solutions of the perturbed problem to the analytical solution of the original (unperturbed) problem on a one-dimensional domain, and we obtain the already mentioned behavior for a fixed mesh. Analogous results are obtained on a square domain with two-dimensional settings for eigenvalues. Moreover, the Laplace and Steklov eigenproblems are considered on regular polygons which are inscribed in the unit disc with an increasing number of sides. For the Laplace eigenvalue problem, we numerically show that the eigenvalues on the polygon can be represented in terms of inverse powers of the number of its sides, as given in [2] (see also [3]). Similar numerical results are shown to be valid for the Steklov eigenvalue problem with regards to several characteristic properties which are not available in the open literature to the best of our knowledge.

Keywords: Laplace EVP, Steklov EVP, FEM, boundary variations.

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Vukman's theorem for symmetric generalized semi-biderivations

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Abstract

In the present note we introduce the notion of symmetric generalized semi-biderivations on rings and prove some basic commutativity results for semi-biderivations. Moreover, our main objective is to extend the theorem of Vukman [2] for biderivation to the case of symmetric generalized semi-biderivations on prime ring. Also we explore some counter examples in favor of the hypothesis of our theorems.

Keywords: Prime ring, generalized bi-derivation, semiderivation, left (right) bi-multiplier.

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Hidden Bifurcation to Multiscroll Chaotic Attractors Via Transformations

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Abstract

In this paper a novel method revealing hidden bifurcations in the multispiral generated by Transformations. The method to find such hidden bifurcation is similar to the method introduced by Menacer, et al. (2016) for Chua multiscroll attractors. We study completely the multispiral Chua system, generated via Transformations, and check numerically our method for numbers of scrolls from 1 to 6.

Keywords: Hidden bifurcation, linearization method, saturated function series

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Spectral properties of the problem on vibrations of a loaded string in weighted grand Lebesgue spaces

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Abstract

In the solution of the vibration problem, which has two ends fixed and a load suspended in the middle, a second-order discontinuous differential equation emerges. In this study, we consider in weighted grand Lebesgue space to reach a wider set of solutions to the spectral problem. However, due to the fact that the weighted grand Lebesgue space $L_{p),\rho}(-1,1)$ is not separable, we have expressed a $G_{p),\rho}(-1,1)$ subspace suitable for the problem with the help of the shift operator. For the basicity properties of the system of eigen and associated functions of the second order discontinuous differential operator, ths spaces $G_{p),\rho}(-1,1)\oplus\mathbb{C}$ and $G_{p),\rho}(-1,1)$ with a general weight function $\rho(\cdot)$ satisfying the Muckenhoupt condition are studied.

Keywords: weighted grand-Lebesgue space, discontinuous spectral problem, basicity, Muckenhoupt condition.

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On Atomic Decomposition with respect to Exponential System in Weighted Morrey-type Spaces

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Abstract

Exponential system in weighted Morrey spaces $L_{p,\lambda}$ is considered in this work. Under natural conditions on the weight function, these spaces are non-separable. Based on the shift operator, we define the subspace $M_{\rho}^{p,\lambda} \subset L_{\rho}^{p,\lambda}$, where infinitely differentiable functions are dense. We consider a case where the weight function ρ does not satisfy the Muckenhoupt condition $A_{p,\lambda}$. We prove that in this case the system is defective, but is not the atomic decomposition of the space $M_{\rho}^{p,\lambda}$.

Keywords: Morrey space, exponential system, atomic decomposition

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A Decomposable Curvature Tensor on the Recurrent Riemannian Space

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Abstract

In this work, we have studied the recurrent Riemannian space with a semi-symmetric metric connection and the curvature tensor decomposed in the form $R^i_{jkl} = v^i \varphi_{jkl}$ where v^i is a contravariant vector field is and φ_{jkl} is a covariant tensor field.

For the space RV_n that has a decomposable curvature tensor in the form $R^i_{jkl} = a^i_j \varphi_{kl}$, we Show that tensor fields a^i_j and φ_{kl} are recurrent. And later we proved two main theorems concerning such spaces.

Keywords: Semi-symmetric metric connection, recurrent Riemannian space, recurrent tensor, decomposable curvature tensor.

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Global existence of solutions for a coupled viscoelastic wave equation with degenerate damping terms

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Abstract

In this talk, we investigated a nonlinear system of viscoelastic equation with degenerate damping and source terms in abounded domain.Under appropriate assumptions on the parameters, degenerate damping terms and the relaxation functions. Then we investigated global existence of solutions.

Keywords: Global existence, relaxation functions, degenerate damping.

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Blow up of solutions for a nonlinear Kirchhoff-type wave equation with degenerate damping terms

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Abstract

In this talk, a finite time blowup of the solutions for a Kirchhoff-type wave equations with degenerate damping terms is considered. We prove finite time nonexistence of weak solutions with positive initial energy.
Keywords: Blow up, Kirchhoff-type, degenerate damping, wave equation.

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On Solvability of Rieman Boundary Value Problems in Hardy-Orlicz Classes and Applications to Basis Problems

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Abstract

This work deals with the Orlicz space and the Hardy-Orlicz classes generated by this space, which consist of analytic functions inside and outside the unit disk. The homogeneous and non-homogeneous Riemann boundary value problems with piecewise continuous coefficients are considered in these classes. New characteristic of Orlicz space is defined which depends on whether the power function belongs to this space or not. Relationship between this characteristic and Boyd indices of Orlicz space is established. The concept of canonical solution of homogeneous problem is defined, which depends on the argument of the coefficient. In terms of the above characteristic, a condition on the jumps of the argument is found which is sufficient for solvability of these problems, and, in case of solvability, a general solution is constructed. An orthogonality condition is given for solvability of non-homogeneous problem. The obtained results are applied to establish the basicity of a linear phase exponential system for Orlicz spaces.

Keywords: Orlicz space, Hardy-Orlicz classes, Riemann boundary value problem, basicity

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Numerical Analysis of Volterra Integral Equations Utilizing Bernstein Type Approximation Technique

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Abstract

In this presentation, we construct a numerical scheme to solve first and second kind of Volterra integral equations with the help of new modification of Bernstein approximation technique which fix the exponential function. In order to validate introduced method, we also present the convergence analysis of this numerical scheme. Finally, we provide the numerical experiments of proposed method to show that its superior properties.

Keywords: Volterra Integral Equation, Bernstein Approximation, Exponential Functions.

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On New Modification of Gamma Operators; Theory and Application

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Abstract

The present paper deals with new modification of Gamma operators preserving polynomials in Bohman-Korovkin sense and study their approximation properties: Voronovskaya type theorems, weighted approximation and rate of convergence are captured. The effectiveness of newly modified operators according to classical ones are presented in certain senses as well. Numerical examples are also presented, highlighting the performance of the new constructions of Gamma operators in the context of one dimensional approximation.

Keywords: Voronovskaya type theorems, Weighted approximation, Rate of convergence.

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Commutative ideals of BCK-algebras based on fuzzy soft set theory

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Abstract

In this paper, further properties of fuzzy soft ideals over BCK/BCI-algebras are investigated. The notion of fuzzy soft commutative ideals over BCK-algebras is introduced, and related properties are investigated. Relations between fuzzy soft ideals and fuzzy soft commutative ideals are discussed, and conditions for a fuzzy soft ideal to be a fuzzy soft commutative ideal are provided. The "AND" operation, extended intersection and union of fuzzy soft (commutative) ideals are dealt with, and characterizations of fuzzy soft (commutative) ideals are considered.

Keywords: BCK/BCI-algebra, (Commutative) ideal, Fuzzy (commutative) ideal, Fuzzy soft set, Fuzzy soft (commutative) ideal.

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Study on a Fuzzy Logic based system using Quality Attributes

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Abstract

Fuzzy Logic is a generalization of standard logic, in which a concept can possess a degree of truth anywhere between 0 and 1. This system is based on fuzzy logic and it suggests the best college for a particular student based on quality attributes. Quality attributes are the factors whose opinions are stated linguistically. We have provided the data of the colleges to the system. System asks for student details and based on these details, system filters the colleges. Then it ranks the filtered colleges based on quality attributes and assigns the best college to student. This system acts as an advisor/mentor and helps students to make good decisions about college selection.

Keywords: Fuzzy Logic, Quality Attributes, Fuzzy Hamming Distance, Distance Matrix.

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Openness and closedness of Measures

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Abstract

In this talk I will give a survey on some results about measures on effect algebras. Effect algebras (alias D-posets) have been independently introduced in 1994 by D.J. Foulis and M.K. Bennett and by F. Chovanec and F. Kopka for modelling unsharp measurement in a quantum mechanical system. They are a generalization of many structures which arise in Quantum Physics and in Mathematical Economics, in particular they are a generalization of orthomodular posets and MV-algebras and therefore of Boolean algebras.

I will offer a version of the open mapping theorem which states that a finite dimensional sigma-order continuous modular measure defined on a sigma-complete effect algebra has closed range, and it is open whenever it is nonatomic.The problem of openness of classical measures, i.e. of measures defined on Boolean algebras, has been treated by several authors, e.g. by Anantharaman and Garg, Karafiat and Spakowski. In particular, Spakowski proved the openness of finite dimensional-valued sigma-additive nonatomic measures defined on sigma-algebras. Lyapunov gave the first proof that a finite dimensional valued sigma-additive measure defined on a sigma-algebra has a closed range, rediscovered by Halmos with different techniques eight years later; I will present a generalization of this theorem valid for (sigma-order continuous) modular measures on effect algebras. I stress that a (sigma-order continuous) modular measure on a Boolean algebra is precisely a (sigma-additive) measure in the usual sense.

Keywords: Open mapping theorem, Lyapunov theorem, modular measures, effect algebras.

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Veracity and satisfiability condition of state equation of bubble liquid

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Abstract

The calculations has carried out veracity and satisfiability condition of state equation of gas-liquid surround in this article. The dependence of the dimensionless radius of the air bubble, of the pressure in the bubble, of gas temperature on the dimensionless time and analogical dependence for the case when the pressure went down abruptly at room temperature in the water depicted. The results showed that the derived state equations agree well with wellknown formula.
Keywords: state equation, gas bubble, pressure, gas-liquid mixture, temperature, volume concentration.

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Accretive Darboux growth in Minkowski spacetime

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Abstract

It is well known that the geometric methods are used in most fields in natural sciences. Kinematics on curves and surfaces as one of these methods is an essential tool for investigating the growth of some biological objects. In [6], the authors give a mathematical framework to model the kinematics of the surface growth of some biological objects by defining a growth velocity at each point on a spatial generating curve. In this study, a time dependent model for the accretive growth is considered. For this, it is defined a growth velocity in the direction of the Darboux vector at every point on a spatial non-null curve in the three dimensional Minkowski spacetime. Also, several examples and visualisations are given to support the theory.

Keywords: Alternative moving frame, accretive growth, Darboux vector, Minkowski space, timelike helix, spacelike helix.

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Scattering Function of the Quadratically Eigenparameter Depending Impulsive Sturm-Liouville Equations

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Abstract

We shall define the impulsive Sturm-Liouville boundary value problem (ISBVP)

$$-u'' + q(z)u = \xi^2 u, \quad z \in [0,1) \cup (1,\infty), \qquad (1)$$

$$(\hbar_0 + \hbar_1\xi + \hbar_2\xi^2)u'(0) + (\eta_0 + \eta_1\xi + \eta_2\xi^2)u(0) = 0 \quad (2)$$

$$u(1^+) = \delta_{11}u(1^-) + \delta_{12}u'(1^-), \qquad (3)$$

$$u'^{(1^+)} = \delta_{21}u(1^-) + \delta_{22}u'\ (1^-),$$

where ξ is a spectral parameter, δ_{11}, δ_{12}, δ_{21}, δ_{22}, $\hbar_i$, $\eta_i, i = 0,1,2$ are real numbers, $\hbar_2\eta_2 \neq 0$, $\delta_{12} \neq 0$, $\delta_{11}\delta_{22} - \delta_{21}\delta_{12} > 0$ and q is a real valued function that satisfies the following conditions

$$\int_0^\infty (1+z)|q(z)|\,dz < \infty.$$

There are some studies examining the scattering analysis of the impulsive Sturm-Liouville equation $[1-3]$. The difference in our study is that the boundary condition is in the quadratic form with the ξ spectral parameter. This gives the problem a new perspective.

The plan of this paper is as follows. In section 2, we deal with the impulsive Sturm-Liouville operator on the semi axis. We first obtain Jost solution and Jost function of ISBVP (1)-(3), then we get the scattering function by using the Jost function. We also investigate characteristic properties of the scattering function of (1)-(3). In section 3, we define the set of eigenvalues of the operator. Furthermore, we get the asymptotic equation for Jost function and resolvent operator of this problem. In section 4, we take an example to demonstrate the validity of the methods and theorems we have shown.

Keywords: Jost solution, Jost function, Impulsive operator, Sturm-Liouville equation, eigenvalue, spectral parameter.

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Effects of Neutrosophic Binomial Distribution on Double Acceptance Sampling Plans

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Abstract

Acceptance sampling plans (ASPs) offers to inspect a small set instead of all outputs in a production process. This approach minimizes the inspection cost dramatically and guarantees the output quality within a predefined risk ratio based on a small sample size. One type of ASPs named double sampling plans (DSPs) gives an ability to minimize the effect of the randomness on inspection results and reach a lower risk level with a small sample size. We know that the ASPs use certain values while formulation and application procedures. However, it is also clear that quality metrics and quality specifications may not be certain in some real cases and they include some vagueness. The fuzzy set theory (FST) is one of the most popular techniques to model the uncertainty in the engineering problems. Additionally, we know that the fuzzy extensions such as Neutrosophic sets (NSs) bring some advantages to manage these uncertainties. Generally, fuzzy DSPs is offered in the literature but it is formulated with α-cut approach to convert the problem into interval valued set problem. With the help of this conversion, it is enough to solve the problem with certain values for the upper and the lower limits of the intervals. However, the uncertainty is generally more complex in real life applications including human factor. NSs that include three terms, truthness (t), indeterminacy (i) and falsity (f) and cover inconsistent data cases are good representation of human thinking under uncertainty. In this study, double acceptance sampling plan is formulated and analyzed based on NSs by using binomial distribution. A numerical example is also presented.

Keywords: Acceptance double sampling plans, Fuzzy sets, Neutrosophic sets, Binomial distribution.

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On some spectral properties and trace formula of differential operator with unbounded operator coefficient

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Abstract

In this article we research asymptotic eigenvalue distrubtion of boundary value problem dependent on spectral parameter in the Hilbert space. Further we calculate the first trace formula of the same problem.

Keywords: Hilbert space, eigenvalue parameter, the first regularized trace

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Homotopy Perturbation Elzaki Transform Method to Random Component Partial Differential Equations

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Abstract

In this study, the series solution of the random component nonlinear partial differential equations are analyzed by using Homotopy Perturbation Elzaki Transform Method (HPETM). The parameters and the initial conditions of the random component nonlinear partial differential equations are studied by Normal distribution. A few examples are indicated to illustrate the effect of the solutions obtained with Homotopy Perturbation Elzaki Transform Method (HPETM). Also, the functions for the expected values and variances of the approximate analytical solutions of the random component nonlinear partial differential equations are acquired in the MAPLE software. Homotopy Perturbation Elzaki Transform Method is applied to analyze the solutions of random component nonlinear partial differential equations. MAPLE software is used for the finding the solutions. Besides, MAPLE software is used for the drawing the figures.

Keywords: Random partial differential equation, expected value, homotopy perturbation Elzaki transform method.

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Homotopy Perturbation Elzaki Transform Method to Random Component Partial Differential Equations

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Abstract

In this study, the series solution of the nonlinear time-fractional gas dynamics equation is analyzed by using Homotopy Perturbation Elzaki Transform Method (HPETM). The fractional derivatives are defined in the Caputo sense. An application is solved by using Homotopy Perturbation Elzaki Transform Method (HPETM). Also, the graph of the solution of the nonlinear time-fractional gas dynamics equation is obtained in the MAPLE software..

Keywords: Gas dynamics equation, homotopy perturbation Elzaki transform method, Mittag Leffer function

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Predicting Anemia in Medical Systems Using Artificial Neural Network Models

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Abstract

Since life is a multivariable function and each variable has a varying level of impact in biological equilibrium, this study concentrates on a study based on as many input variables as possible. Therefore, since the evaluation of anemia is difficult or expensive or time consuming with the use of classical approaches, this study aimed to estimate the relationship between various types of blood features and types of anemia over the biomedical environment. To accomplish this, the artificial neural network (ANN) algorithms have effectively been designed for the population of interest. The research is produced in terms of data consisting of 539 subjects provided from blood laboratories and they have been analyzed by using the ANNs with both one hidden layer and two hidden layers. This study gives the best accuracy of the results and is seen to be a very successful first attempt to predict anemia types based on the biophysical features. Thus, the present assessments show that the ANN provides a successful test for predicting anemia types with high accuracy.

Keywords: Artificial neural network, Prediction, Anemia, Blood features, Applied mathematics

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FINITELY g-SUPPLEMENTED MODULES

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Abstract

Let M be an R module. If every …nitely generated submodule of M has a g-supplement in M, then M is called a …nitely g-supplemented (or brie‡y fg-supplemented) module. In this work, some properties of these modules are investigated.

Key words: Essential Submodules, g-Small Submodules, Supplemented Modules, g-Supplemented Modules.

2010 Mathematics Subject Classification: 16D10, 16D70.

Results

Proposition 1 Every g-supplemented module is fg-supplemented.
Proposition 2 Let M be a fg-supplemented R module. If M is noetherian, then M is g-supplemented.
Lemma 3 Let M be a fg-supplemented R module and N be a …nitely generated submodule of M. Then M=N is fg-supplemented.
Corollary 4 Let M be a fg-supplemented R module and N be a cyclic submodule of M. Then M=N is fg-supplemented.
Corollary 5 Let f : M !N be an R module epimomorphism and Kef be …nitely generated. If M is fg-supplemented, then N is also fg-supplemented.
Corollary 6 Let f : M ! N be an R module epimomorphism with cyclic kernel. If M is fg-supplemented, then N is also fg-supplemented.
Lemma 7 Let M be a fg-supplemented R module and N M. Then M=N is fg-supplemented.
Corollary 8 Let f : M ! N be an R module epimomorphism with small kernel. If M is fg-supplemented, then N is also fg-supplemented.

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COFINITELY eg-SUPPLEMENTED MODULES

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Abstract

Let M be an R module. If every co…nite essential submodule of M has a g-supplement in M, then M is called a co…nitely essential g-supplemented (or brie‡y co…nitely eg-supplemented) module. In this work, some properties of these modules are investigated.

Key words: g-Small Submodules, Co…nite Submodules, Essential Submodules, g-Supplemented Modules.

2010 Mathematics Subject Classification: 16D10, 16D70.

Results

Lemma 1 Every essential g-supplemented module is co…nitely eg-supplemented.
Corollary 2 Every g-supplemented module is co…nitely eg-supplemented.
Lemma 3 Every co…nitely g-supplemented module is co…nitely eg-supplemented.
Corollary 4 Let M = $\sum_{i\in I} M_i$. If M_i is co□nitely g-supplemented for every i 2I, then M is co□nitely eg-supplemented.
Corollary 5 Let M be a co□nitely g-supplemented module. Then every M generated module is co□nitely eg-supplemented.
Corollary 6 Let R be a ring. If RR is g-supplemented, then every R module is eg-supplemented.
Proposition 7 Let M be an co…nitely eg-supplemented module. If every nonzero submodule of M is essential in M, then M is co…nitely g-supplemented.

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On an algorithm for controlling the depth of feedback in an optimization problem taking into account the distribution function of perturbations

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Abstract

An algorithm is proposed for generating a feedback function for the task of optimizing quality when performing tasks on the volume of output. The problem is characterized by the presence of an external disturbance factor with a known distribution function.

Keywords: Quality management, optimization based on constraints, stochastic problem.

Product Quality Function Set

$$F = F(x,u) \tag{1}$$

where x - is the quality indicator of processed raw materials - a disturbance with a distribution function $y = \varphi(x)$; management, which is the intensity of current production. The planned production volume G for the period $t \in (0,T]$ is given in the form:

$$\int_0^T u(t)dt = G \tag{2}$$

A rule is set to control the average quality over a period of quality (1), which must be observed at arbitrary points in time $t \in (0,T]$:

$$u[x(t),L(t)] \equiv \arg\max_{\tilde{u} \in U} \left\{ F(x(t),\tilde{u}) - L(t)\left(\tilde{u} - \frac{G - \int_0^t u(\tau)d\tau}{T-t} \right)^2 \right\}. \tag{3}$$

Based on the given distribution function $y = \varphi(x)$, the given weight coefficients of the quality optimality and the attainability of the planned task, it is necessary to determine the optimal feedback depth function $L(t)$ that delivers the maximum

$$E[F(x(t),u(t)] \to \max \tag{4}$$

The algorithm for solving this problem is based on the necessary condition, which consists in the stable maintenance of mathematical expectations of the value of the quality function and the reachability function of the task according to the plan throughout the entire planning period.

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Hermit Operational Matrix for Solving Fractional Differential Equations

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Abstract

This paper aims to solve the fractional differential equations (FDEs) with operational matrix method by Hermite polynomials in the sense of Caputo derivative. For this purpose, we attempt to re-define the FDEs with a set of algebraic equations with initial conditions which simplifies the complete problem. We achieve either exact or approximated solutions by solving these algebraic equations with the proposed method. To indicate the efficiency of the proposed method, various illustrative examples are solved. The main advantage of the method is its high speed which requires only a few number of step for solution. Therefore, the complexity level of the solution is low which makes it practical. We also consider the FDEs with non-polynomials solution making the proposed method more reliable. Another important feature of this work is that there is a big gap in literature for Hermite operational matrix which is fulfilled by this work.

Keywords: Fractional integration, Hermite polynomials, operational matrix, Caputo.

Nonexistence of Solutions of a Delayed Wave Equation with Variable-Exponents

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Abstract

In this work, we consider a nonlinear delayed wave equation with variable-exponents. Under suitable conditions, we prove the blow-up of solutions in a finite time. Several physical phenomena such as flows of electro-rheological fluids or fluids with temperature-dependent viscosity, nonlinear viscoelasticity, filtration processes through a porous media and image processing are modelled by equations with variable exponents of nonlinearity.

Keywords: Blow-up, Nonlinear Wave Equation, Variable-Exponents, Delay Term.

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Decay and Blow up of Solutions for a Delayed Wave Equation with Variable-Exponents

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Abstract

This work deals with a nonlinear wave equation with variable-exponents and a delay term. Under suitable conditions, we study the blow-up of solutions in a finite time. In the absence of the source term, we prove a decay result. Generally, time delay effects arise in many applications and practical problems such as physical, chemical, biological, thermal and economic phenomena.

Keywords: Blow-up, Decay, Variable-Exponent, Delay.

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A Fourier Transform Technique for Shape Detection of 3-D Rigid Objects

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Abstract

In this work, a fiber optic Lloyd's mirror assembly is used to obtain various optical interference patterns for the detection of 3D rigid body shapes. Using Lloyd's systems, a structured light pattern is projected onto an object surface, and a CCD camera captures the deformed interference pattern. The modulated data is then 2D Fourier transformed into its spatial frequency. The desired data is filtered from this information in the Fourier domain using a bandpass filter, and the surface profile of the object is obtained by applying the inverse 2D Fourier transform and a phase unwrapping algorithm. We anticipate that using a fiber optic Lloyd's mirror assembly makes the interferometric system even more compact, more stable, and much simpler than using the structured light pattern systems previously employed in optical health monitoring.

Keywords: Fourier Transform, Fringe analysis, Interferometry, Surface measurements.

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An extensive statistical study for the Leukemia mathematical model using the RVT technique

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Abstract

This paper provides a probabilistic study for the four compartmental leukemia mathematical model. Our study focuses on the randomized endemic equilibrium state considering the profileration rate of the immune cells due to the cancer relapse is a continuous random variable. This treatment make the presented model to be more realistic and efficient. Depending on the Random Variable Transformation (RVT) technique, the first probability density functions (1-PDFs) for the solution processes of susceptible blood cells, infected blood cells, cancer cells and immune cells are explicitly derived at the equilibrium state. These PDFs are general and valid for any probabilistic distribution of the input random variable. Relying on the obtained PDFs, the main statistical properties, specifically, the mean and the variance functions, as well as the confidence intervals for the solution processes are conducted. To test the validity of the theoretical findings associated to the proposed randomized leukemia model, some numerical results presented through an illustrative example.

Keywords: leukemia mathematical model, Random Variable Transformation (RVT) technique, First probability density function (1-PDF), randomized endemic equilibrium state, random profileration rate, adoptive immune cells therapy.

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An Assessment On Factorization Of Determinants

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Abstract

This study aims to examine different teaching methods for the subject of determinants, which are taught in the linear algebra classes, to the first year students of the universities. These methods were evaluated according to the answers given by the students to the questions. For this purpose, two groups, group A and B, each consisting of 70 students, were selected. Students in Group A and B were asked to calculate the determinant in concrete form and to find the factorization of the determinants in abstract form. It has been observed that the group B students were more successful.. Findings of the research will be explained in presentation with details.

Keywords: Linear algebra, teaching linear algebra, factorization of determinant of matrix

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Investigation of Γ −Invariant Equivalence Relations of Modular Groups and Subgroups

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Abstract

In the study published by Jones, Singerman and Wicks (1991), the modular group, the movement of an element of the modular group on $\widehat{\mathbb{Q}}$ (extended set of rational numbers) in hyperbolic geometry and, Farey graph, $G_{u,n}$ and $F_{u,n}$ were studied. Also, it is showed that the fixed of any two points is conjugated in Γ, and the element of the modular group that leaves constant an element on $\widehat{\mathbb{Q}}$ is infinite period. So, the element of the modular group that leaves the ∞ element constant is symbolized as Γ_∞. In the same study, H, the subgroups of Γ of containing Γ_∞ are obtained and invariant equivalence relations are generated on $\widehat{\mathbb{Q}}$. In this study, we show that, the element that fixed $\frac{x}{y}$ in modular group according to based on the choice of $\frac{x}{y}$ for $x, y \in \mathbb{Z}$ and $(x, y) = 1$, instead of a special value of set $\widehat{\mathbb{Q}}$, such as ∞. Similarly, we study subgroups H containing $\Gamma_{\frac{x}{y}}$ and we examine that the invariants equivalence relations on $\widehat{\mathbb{Q}}$.

Keywords: Modular group, Infinite period, Invariant equivalence relations.

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Obtaining Some Identities with the n^{th} Power of Matrix $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ Under The Lorentzian Matrix Product

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Abstract

The Fibonacci number sequence and related calculations come up in scientific facts in many events we encounter in daily life. This special number sequence is processed in the occurrence of many events such as calculating the diameter of the equatorial circumference of the Earth, flowers, growth and structures of leaves, trees, reproduction of bees, sunflower and so on. (Koshy, 2001). However, in recent years, the relation between the Fibonacci and Lucas Number sequences with continued fractions and matrices has intensively been studied. Many identities have been found by some 2×2 types of special matrices with n^{th} power that have been associated with the Fibonacci and Lucas numbers. The aim of this study is to examine matrix $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ under the lorentzian matrix product with n^{th} power, quadratic equations and characteristic roots unlike the classical matrix product. In addition, we want to acquire some identities with the help of matrix $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$ under the lorentzian matrix product with n^{th} power in relation to the Fibonacci and Lucas numbers.

Keywords: Fibonacci and Lucas Numbers, Lorentzian Matrix Multiplication, Quadratic Equation, Characteristic Root.

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Some Properties of Internal State Operator on Sheffer Stroke Basic Algebras

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Abstract

In this study, we introduce an internal state operator on Sheffer stroke basic algebras. We handle some characteristic properties of this operator and give some examples about these properties. Moreover, we construct a relation between orthomodular lattices and Shefer stroke basic algeras by the help of the internal state operator.

Keywords: Internal state operator, Sheffer stroke basic algebras, Orthomodular lattices

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Lemniscate and Exponential Starlikeness of Regular Coulomb Wave Functions

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Abstract

In this study, a normalized form of regular Coulomb wave function is considered. By using differential subordination method due to Miller and Mocanu, we determine some conditions on the parameters such that the normalized regular Coulomb wave function is lemniscate starlike and exponential starlike in the open unit disk $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$, respectively. In additon, by using the relationship between regular Coulomb wave function and Bessel function of the first kind we give some conditions for which the classical Bessel function of the first kind is lemniscate and exponential starlike in the unit disk $\mathbb{D}$.

Keywords: Analytic function, Coulomb wave function, Exponential starlikeness, Lemniscate of Bernoulli, Lemniscate starlikeness, starlike functions.

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Mathematical Modeling of Line Balancing Problem

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Abstract

Simple assembly line balancing (SALB) is an approach for assignment of the tasks to the various workstations along the assembly line so that the precedence relations are satisfied and some resource constraints considered. In this work, a multiobjective mathematical model proposed using Type-E assembly line problem, is one of the simple assembly line balancing problem. Unlike Type-1 and Type-2 assembly line balancing problems, only a few studied on simple assembly line balancing of Type-E problem (SALB-E) because of its complexity. A mathematical model is developed for increasing the efficiency of an aircraft component production. The aim of the mathematical model is minimizing the smoothness index of the overall assembly line and each assembly station, and manpower costs. Most aircraft assembly lines are labor-intensive assembly lines. The mathematical model of personnel flow and assembly line balancing are formulated based on features of the aircraft component assembly line. The solution obtained improves assembly production efficiency and also reduces idle time.

Keywords: Simple assembly line balancing, Type-E, Workload smoothing, Balancing efficiency, Cycle time, Assignment, Mathematical model

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Some properties of convolution in symmetric spaces and approximate identity

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Abstract

This work deals with symmetric space of functions and its subspace where continuous functions are dense. Main properties of convolution are established in this space. It is proved that the convolution can be approximated by linear combinations of shifts in a subspace of this space. Approximate identity is also considered in that subspace.

Keywords: symmetric spaces, convolution, approximate identity
Classification 2010: 46E30, 30E25

Definition 1. *Let* ρ *be a function norm. The collection* $X = X(\rho)$ *of all functions* f *in* $\mathcal{M}$ *for which* $\rho(|f|) < +\infty$ *is called a Banach function space. For each* $f \in X$, *define* $\|f\|_X = \rho(|f|)$.

Definition 2. *Let X be a Banach function space. The closure of the set of simple functions* $\mathcal{M}_s$ *in* X *is denoted by* X_b.

Theorem. *Let* X *be a r.i.s. with Boyd indices* $\alpha_X; \beta_X \in (0,1)$. *Let* $f \in L_1(-\pi,\pi)$ *and* $g \in E$, *where* E *denotes any one of the spaces* $C[-\pi,\pi]$ *or* X_b. *Then the convolution* $f * g$ *in* E *can be approximated by finite linear combinations of shifts* g, *i.e.* $\forall \varepsilon > 0$, $\exists \{a_k\}_1^n \subset [-\pi,\pi] \wedge \{\lambda_k\}_1^n \subset R$:

$$\left\| f * g - \sum_{k=1}^{n} \lambda_k \mathrm{T}_{a_k} g \right\|_E < \varepsilon .$$

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Some Observations on Generalized Non-expansive Mappings and Convergence Results

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Abstract: In this talk, we show that the classes of generalized non-expansive mappings due to Hardy and Rogers and the mappings satisfying Suzuki's condition *(C)* are independent and study some basic properties of generalized non-expansive mappings. Further, we introduce a new iterative scheme, called JF iterative scheme, and prove convergence results for generalized non-expansive mappings due to Hardy and Rogers in uniformly convex Banach spaces. Moreover, we show numerically that JF iterative scheme converges to a fixed point of generalized non-expansive mappings faster than some known and leading iterative schemes. As an application, we utilize newly defined iterative scheme to approximate the solution of a delay differential equation. Also, we present some nontrivial illustrative numerical examples to support main results.

Finally, we also approximate common fixed points of the generalized non-expansive mapping via one step iterative scheme in uniformly convex Banach space. We utilize the result to solve image recovery problem in Banach space. Some examples are furnished in the support of the results.

Keywords: Generalized non-expansive mappings ▪ Suzuki's condition (C) ▪ Fixed points ▪ JF iterative scheme ▪ uniformly convex Banach spaces ▪ Image recovery problem.

Mathematics Subject Classification: 47H09 ▪ 47H10

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NATURAL TRANSFORM ADOMIAN DECOMPOSITION METHOD(NTADM) FOR EVALUATION OF TWO DIMENSIONAL FRACTIONAL PARTIAL DIFFERENTIAL EQUATION, AN APPLICATION TO FINANCIAL MODELLING:

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Abstract

This paper represents the new novel technique of Natural Transform Adomian Decomposition Method (NTADTM) to get real and Imaginary option prices of two stocks in form of analytic infinite series by solving Schrödinger Black Schloes time fractional ordered PDE consisting two different stocks. This technique is the combination of Natural Transform and Adomian Decomposition Method. Accordingly, the technique is found to be appropriate for financial models that can be expressed as partial differential equations of integer and fractional orders, subjected to initial or boundary conditions..

Key words: Natural Integrals Transform , Adomian Decomposition Method, Time fractional order Schrödinger black Scholes PDE, Options, Fractional Order Caputo Calculus, Matlab codes and programming.

Regionalization of precipitation zones by GWPS

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Abstract

Regionalization as for precipitation is crucial in locating the area for new dams, area for new supplies for both developed and urban employments. It permits the confinement of catchment regions of related precipitation highlights and appearances the framework elements in the zone, so giving complete knowledge about the precipitation. Precipitation areas were perceived inside Indus River , by looking at the precipitation frequencies by methods for global wavelet power spectrum. Information from 52 rain gauges (Pakistan Metrological Department and Pakistan Statistical Bureau are the fundamental sources of information) were considered and the impacts of the GWPS demonstrated distinctive recurrence everywhere throughout the basin of couple of regions; notwithstanding, different frequencies are existing with somewhat essentialness that shows changes in the precipitation framework. In spite of the fact that deciding the GWPS existing at a yearly recurrence, they demonstrated peculiar rainy patterns were identified (indicated A and B) that could be utilized to portray the area. Hence, the five sub-areas with predictable stormy examples were perceived as Region (1, 2, 3, 4 and 5) with frequency patterns A and B, the climatic conditions of the study area in terms of precipitation are extremely diverse in these regions.

KeyWords: Wavelet, Spectrum, GWPS, power spectrum, regionalization

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Control of PID Parameters by Iterative Learning Based on Neural Network

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Abstract

Iterative learning refers to the development, analysis and implementation of methods which allow a machine to evolve through a learning process, and thus perform tasks that are difficult or impossible to perform by more conventional algorithms.Learning is a dynamic and iterative process used for modifying the parameters of a network in response to the stimulus it receives from its environment. The learning type is determined according to parameter changes. Our contribution in this article is the design and development of an algorithm that can optimize the parameters of a PID controller for the control of repetitive systems, using the iterative learning approach based on neural network. Simulation is conducted to assess the sufficiency of our approach

Keywords:
learning process, PID controller, dynamic process, neural network.

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On Some Relations and Applications of the $\mathcal{M}_{v,n,2}$-Integral Transform

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Abstract

In the present paper, the authors introduce two new integral transforms,

$$\mathcal{M}_{v,n,2}\{f(x);(y,w)\} = \int_0^\infty \frac{xe^{-y^2x^2}}{(x^{2n}+w^{2n})^v} f(wx)dx,$$

where $Re(v) > 0, n \in \mathbb{Z}^+$ and

$$\mathcal{N}_2^+\{f(x);(y,w)\} = \int_0^\infty xe^{-y^2x^2} f(wx)dx, \quad y > 0, w > 0.$$

Several theorems that are dealing with general properties of the $\mathcal{M}_{v,n,2}$-integral transform are proved. The existence theorem, convolution theorem and inversion theorem for the $\mathcal{M}_{v,n,2}$-integral transform are given. Generalized Natural transform $\mathcal{N}_2^+$, which is a special form of $\mathcal{M}_{v,n,2}$-transform are used for to find solutions of an initial-boundary value problem and an integral equation.

Keywords: Laplace transform, Natural transform, .Sumudu transform, H-function, H-transform

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On the Formalization of McShane Integral in the HOL4 Theorem Prover

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Abstract

McShane integral is a type of gauge integral that differs a little from the definition of the Henstock-Kurzweil integral. For instance, a significant difference between the two is that every McShane integrable function is Henstock-Kurzweil integrable, but the opposite is not necessarily true. Besides, unlike the Henstock-Kurzweil integral, the Lebesgue integral can be represented by using McShane integral, and these two integrals coincide on Euclidean spaces. Furthermore, the McShane integral provides the advantages of the Lebesgue integral without the need of the measure theory. These integrals can be used in various fields of science and engineering's like reliability analysis and probability theory. In this study, we aim to formalize the McShane integral by developing proper definitions and proofs of its basic properties using the HOL4 theorem prover, which already contains many useful libraries (theories) such as Lebesgue integral, real, arithmetic, and derivate.

Keywords: McShane integral, Lebesgue integral, Henstock-Kurzweil integral, gauge, γ-fine free tagged partition, Higher-order logic, HOL4 teorem prover

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A Brief Introduction to Henstock-Kurzweil Integral

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Abstract

Henstock-Kurzweil integral, known as gauge integral or generalized Riemann integral, is defined based on the modified Riemann sum. It extends the Lebesgue integral so that it has a wider class of integrable functions. That is, the Lebesgue integrability implies Henstock-Kurzweil integrability, however the converse is not necessarily true. Furthermore, it has some applications in mathematics, computer science and engineering. In this work, we examine the Henstock-Kurzweil integral and present its fundamental properites and some examples, especially functions that are Henstock-Kurzweil integrable but not Lebesgue integrable. Finally, we study the relationship between Henstock-Kurzweil integral and Lebesgue integral.

Keywords: Henstock-Kurzweil integral, Generalized Riemann integral, Gauge integral, Lebesgue integral, gauge, tagged partition, Henstock's lemma

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A CONVEXITY PROBLEM FOR A SEMI-LINEAR PDE

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Abstract

In this paper we prove convexity of super-level sets of a semi-linear PDE with a non-monotone right hand side, and with a free boundary

$$(u\Delta u \;=\; \chi\{0 < u < 1\}\; in\; Rn\backslash D,$$

$$u = \;2\; on\; \partial D.$$

Here D is assumed to be convex, and $n \geq 2$. The main difficulty of this problem is that the right hand side is non-monotone and no a priori regularity is known about the boundary $\partial\{u > 0\}$.

Keywords: Convexity, starshapedness, free boundary

2010 Mathematics Subject Classification:[2000]35R35

PRECISE MORREY REGULARITY OF THE WEAK SOLUTIONS TO A KIND OF QUASILINEAR SYSTEMS WITH DISCONTINUOUS DATA

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Abstract

We consider the Dirichlet problem for a class of quasilinear elliptic systems in domain with irregular boundary. The principal part satis_es componentwise coercivity condition and the nonlinear terms are Carath_eodory maps having Morrey regularity in x and verifying controlled growth conditions with respect to the other variables. We have obtained boundedness of the weak solution to the problem that permits to apply an iteration procedure in order to _nd optimal Morrey regularity of its gradient.

On Some Integral Type Inequality on Time Scales

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Abstract

Time scales have been in the study area of many mathematicians for the last 30 years. Some of these studies are inequalities and dynamic equations. And also, inequalities and dynamic equations contributed to the solution of many problems in various branches of science. In this study, some integral type inequalities are presented in two dimensional case on time scales via delta integral.

Keywords: Integral type inequality, Time scales, Integral inequalities.

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An Investigation on Fractional Maximal Operator in Time Scales

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Abstract

Dynamic equations, operators and inequalities have recently increased their motivating role in time scales. Time scales have been the field of study of many mathematicians and scientists working in different sciences for the last 30 years. In this research, we will prove that, for $1 < p(x) < \infty$, the variable exponent $L^{p(.)}$ norm of the restricted centered fractional Maximal delta integral operator $M^c_{a,\delta}$ equals the norm of the centered fractional Maximal delta integral operator M^c_a for all $0 < \delta < \infty$.

Keywords: Time scales, variable exponent, fractional Maximal operator.

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Optimal function spaces for the Laplace transform

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Abstract

We investigate the action of the classical Laplace integral transform $\boldsymbol{L}$, where $\boldsymbol{Lf(x)} = \int_0^\infty \boldsymbol{e^{-st}f(t)dt}$, on rearrangement invariant (r.i.) function spaces on (0,∞). Primary attention is given to the optimality of the range and the domain spaces. Thus, it is proved that given an r.i. function space X such that the associated r.i. function space X′ contains the function min(1,1/t), then there exists an optimal r.i. function space Y for which the Laplace transform $\boldsymbol{L}: \boldsymbol{X} \to \boldsymbol{Y}$ is bounded. On the other hand, when the function min(1,1/t) does not belong to X′, then there does not exist an r.i. space Y for which $\boldsymbol{L}: \boldsymbol{X} \to \boldsymbol{Y}$ is bounded. Similar results are stated for the optimality of the domain when the r.i. range space is fixed. Applications are given for the special scale of Lorentz spaces $\boldsymbol{L^{p,q}}$. Thus for 1<p<∞, p′=p/(p−1) is the conjugate index, and q∈[1,∞], the Laplace transform $\boldsymbol{L}: \boldsymbol{L^{p,q}} \to \boldsymbol{L^{p',q}}$ is bounded, and both the domain space and the target space are optimal.

Keywords: Laplace transform, Rearrangement-invariant spaces, Optimal domain, Optimal range, Lorentz spaces

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Venttsel boundary value problem with discontinuous data

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Abstract

We study discontinuous Venttsel boundary value problem for linear second-order elliptic equations with VMO coefficients. It is obtained strong solvability in Sobolev spaces with optimal exponent ranges.

Keywords: Venttsel problem, VMO, strong solvability.

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Hyers-Ulam-Rassias Stability of the First Order Nonhomogeneous Linear Dynamic Equation on Time Scale

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Abstract

In this study, the method used by Jung [1] was first used to show the Hyers-Ulam-Rassias stability of the first order nonhomogeneous linear dynamic equation on the time scale. The method used for the Hyers-Ulam-Rassias stability of this equation was performed for the closed and limited $[a,b]_{\mathbb{T}}$ range of real numbers set.

Keywords: Hyers-Ulam-Rassias stability, time scale, first order nonhomogeneous linear dynamic equation.

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Mathematical simulation models for plantain Moko disease

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Abstract

Moko is a disease of the plantain crop that has caused great economic losses and currently continues without proper management. The use of quantitative methods based on mathematical simulation models has gained importance to devise effective control programs and interpret epidemiological patterns. For this purpose, it was proposed to determine the appropriate prevention strategies for the incidence of plantain Moko disease, using a simulation model. A population simulation model was presented with nonlinear ordinary differential equations varying disease prevention scenarios with a population of susceptible and infected plants over time. A crop with a variable population of plants and a logistical replanting growth were assumed taking into account the maximum capacity of plants in the delimited study area. The simulations were carried out in the Maple 18 software, the propagation threshold and the sensitivity analysis were determined considering $(\boldsymbol{f})$ the prevention strategies used (disinfestation of tools and footwear, pruning of weeds, among others) and the elimination of infected plants $(\boldsymbol{g})$, observing by determining the threshold and its respective sensitivity analysis that the parameter that most influences the threshold value is $(\boldsymbol{g})$. However, to decrease production costs due to the high implementation of prevention strategies, different scenarios are shown that favor the control of the disease and decrease these costs.

Keywords: mathematical models; moko; plantain; *Ralstonia solanacearum*; prevention; threshold.

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Minimum Generating Set and Rank of $N(C_n)$

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Abstract

Let C_n be the semigroup of all order-preserving and decreasing transformations on $X_n = \{1, \dots n\}$ under its natural order, and let $N(C_n)$ be the subsemigroup of all nilpotent elements of C_n. In this paper we determine the minimum generating set of $N(C_n)$ and so the rank of $N(C_n)$.

Keywords: Order-preserving/decreasing transformation, nilpotent semigroup, minimum generating set, rank.

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The Number of m-nilpotent Elements in $N(C_n)$

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Abstract

For $n \in \mathbb{N}$, let C_n be the semigroup of all order-preserving and decreasing transformations on $X_n = \{1, \dots n\}$, under its natural order, and let $N(C_n)$ be the set of all nilpotent elements of C_n and Fix(α)=$\{x \in X_n : x\alpha = x\}$ for any transformation α. An element of a finite semigroup is called m-potent (m-nilpotent) element if $a^{m+1} = a^m$ ($a^m = 0$) and $a, a^2, \dots, a^m$ are distinct. In this paper we optain a formulae for the number of m-nilpotent elements and so the number of m-potent elements in $N(C_n)$ for $1 \leq m \leq n-1$.

Keywords: Order-preserving/decreasing transformation, m-nilpotent.

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Regression analysis based on stress tests and human errors

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Abstract

Stress tests are the essencial part of the Solvency II insurance regulation system. However, the results of stress tests are the dependent of human errors based on the application of stress tests. It will be measured by the regression analysis about relationthip of human error and truthfulness, integrity of stress tests.

Keywords: Regression, linear regression, multiple regression analysis

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An Application of Müntz Wavelets Galerkin Method for Solving the Fractional Differential Equations

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Abstract

In this study, the Müntz wavelet which is a one of the quitely fresh wavelet types is considered to derive a solution of the fractional differential equations in Caputo sense. The Müntz wavelets can be differently expressed using Müntz-Legendre polynomials [1], Jacobi polynomials [2] or modified Müntz formula [3]. The operational matrices for fractional integration and derivative operators are given for Müntz wavelets [4]. The basis of wavelets is a powerful tool for solving integral and differential equations. The main difference between Haar, Legendre, Chebyshev wavelets and Müntz wavelets is in their degree of the extended sentences. Since the degrees of the Müntz polynomials are complex, Müntz wavelets both present a high accuracy for all complex functions or functions with fractional powers and also cover a broad range of functions primarily occuring in fractional models. The one of the advantages of the method is that the used method enables a simple procedure to convert the differential or integral equations to an algebraic system to be simply solved by many conventional methods in the literature. The procedure for solving equation is tested on some examples to show the applicability, efficiency and accuracy of the used method. The numerical computations in this study are done by using Maple software. The values obtained from the solution of the considered equation by Müntz wavelet method are compared with the other numeric and exact solution in the literature. The comparison of approximate and analytic solution of the problem are visualized by graphics and the errors between the solutions are comparatively shown in the tables. The numerical findings show that the method is quitely effective since it has easy algorithm, high accuracy, less computational complexity and less CPU time for solving the considered equation.
Keywords: Fractional differential equations, Müntz wavelet, numeric methods

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The first author would like to thank Scientific and Technological Research Council of Turkey (TUBITAK) for a financial support of 2211-A Fellowship Program.

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On the Sinc-Galerkin Method for Solving Fractional Integro-differential Equations

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Abstract

In this paper, numerical solution of some class of fractional integro-differential equations is considered. By using Sinc-Galerkin method, the considered equations are replaced by a system of linear algebraic equations. Some examples are given to show the ability of the used method. The numerical findings show that the method is pretty effective and it has easy algorithm, high accuracy.
Keywords: Fractional integro-differential equations, sinc method

Acknowledgements
The first author would like to thank Scientific and Technological Research Council of Turkey (TUBITAK) for a financial support of 2211-A Fellowship Program.

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Evaluating the Smart Campus Key Factors with Multi-Criteria Decision Making Approach under Intuitionistic Fuzzy Environment

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Abstract

Universities are institutions where high-level education and training activities are offered and scientific researches and publications are made. These institutions are complex due to activities such as resource management, organization of various activities, training of people; so innovatory and multifunctional approaches should be used to deal with the problems that may be encountered in universities due to these features [1]. The administration of many students, activities and services is one of the biggest hardship that requires improving processes in the concept of smart campus [2]. Smart campus is the combination of smart technologies and physical infrastructure for educational institutions' advanced technologies, decision making, campus sustainability [3] and can be designed to realize multi-purpose uses such as integrated environment, learning and life based on Internet of Things (IoT) [4]. Smart campus features include extensive environmental awareness, uninterrupted networking, big data support, open learning environment and personalized services for teachers and students [5]. Within the scope of the smart campus, various applications such as smart microgrid, smart classrooms, controlling the visual and thermal properties of buildings, face recognition / smart cards have been performed [3]. The concept of smart campus can be considered as an adaptation of the smart city model to university campuses [1]. Despite the differences in size and type structures, university campuses can be compared with cities so that the smart city model can be used to create a smart campus [6]. It can be claimed that the smart city model is eligible to be transformed in the smart campus model, as universities have comparable problems such as environmental impact, management and organization problems, internal and external mobility and infrastructures, low efficiency and lack of basic services and features, such as cities [6]. Smart city model is based on six smart axes: governance, people, economy, environment, life and mobility [7]. These key indicators can be used to determine the actions to be taken and the strategies to be applied to solve the problems encountered in the smart campus models which are already used to solve the problems in the smart city models. For this purpose, we conducted an evaluation study to determine the importance rankings of "Governance, People, Economy, Environment, Life and Mobility" criteria, which are the key factors that can be adopted in determining the actions and strategies to be implemented in smart campus applications. The evaluation of the criteria was handled within the scope of multi-criteria decision making analysis and in this context, AHP (Analytic Hierarchy Process) method, one of the most used multi-criteria decision-making methods, was used to determine the importance of criteria. At this point, we utilized fuzzy logic to convert linguistic variables to be used in calculations into numerical expressions. We even applied to the Intuitionistic Fuzzy Sets which is one of the extended version of regular fuzzy sets to reflect uncertainties in linguistic expressions better. Thus, it was determined which criteria should be given more importance in the selection of the most appropriate strategies in smart campus applications. In future studies, in line with these determined criteria and their weights, the problem of strategy selection for smart campuses can be handled as a multi criteria decision making problem and even comparative analysis can be performed by applying to different fuzzy sets extensions.

Keywords: Intuitionistic Fuzzy Sets, Key factors, Multi-Criteria Decision Making, Smart campus

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A bioeconomic differential algebraic predator–prey model with harvesting

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Abstract

In this work, we propose a bioeconomic differential algebraic predator–prey model with Holling type II functional response and a harvesting of both prey and the predator populations. By taking into account the economic factor, a bioeconomic differential algebraic equation is obtained.

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - a_1 * x * \frac{y}{n+x} - q_1 * E * x$$
$$\frac{dy}{dt} = sy\left(-1 + a_2 * \frac{x}{n+x}\right) - q_2 * E * y$$
$$0 = q_1 * E(p_1 * x - c_1) + q_2 * E(p_2 * y - c_2) - m$$

The effect of economic profit on the model is investigated. In particular, the stability and bifurcations of the system is obtained by as the economic profit parameter is varied.

MSC 2010: 92D30, 34C23, 34D20

Keywords: Biological economic system, Differential algebraic system, Predator–prey model, Hopf bifurcation

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Computational Analysis of Volterra Integral Equations with Szasz-Mirakyan Type Approximation Method

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Abstract

In this presentation, we introduce a numerical scheme to solve first and second kind Volterra integral equations with the aid of Szasz-Mirakyan type operators which preserve exponential functions.
Keywords: Numerical solution, Szasz-Mirakyan operator, Volterra integral equation.

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Mathematical justification of the obstacle problem in the case of piezoelectric shell

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Abstract

The objective of this work is to study the asymptotic justification of a new twodimensional model for the equilibrium state of a piezoelectric linear shell in frictional contact with a rigid foundation. More precisely, we consider the Signorini problem with Tresca friction of piezoelectric linear shell in contact with a rigid foundation. Then, we establish the convergence of the mechanical displacement and the electric potential as the thickness of the shell goes to zero.

Keyword(s): Asymptotic modeling , Signorini problem , anisotropic , piezoelectric, linear elastic

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On a Second Order Modular Grad-Div Stabilization Method for the Boussinesq Flows

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Abstract

This paper studies a second order modular grad-div finite element stabilization method for approximating solutions of the incompressible non-isothermal flows which uses the Boussinesq approximation. The proposed method adds a minimally intrusive step for the velocity into an existing Boussinesq code. This intrusive step penalizes the divergence of the velocity error both in L^2 and H^1-norms and is decoupled from the evolution equations. The paper provides unconditionally stability and convergence results without their proofs. Numerical experiments confirm theoretical convergence rates, and show that MGD-FEM has a similar positive effect on Marsigli's experiment as in the usual grad-div stabilization.

Keywords: Modular grad-div, finite element method, the Boussinesq equations.

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Remark on the Yosida approximation iterative technique for split monotone Yosida variational inclusions

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Abstract

In an attractive article Rahman et al. introduced the split monotone Yosida variational inclusions (SMYVI) and estimate the approximate solution of the split monotone Yosida variational inclusions using nonexpansive property of operators. The main result of this paper has aw and not correct in the present form. We modify the SMYVI and give the strong convergence theorem under some new assumptions. We also give a weak convergence theorem to solve modied split Yosida variational inclusion problem using property of averaged operator with three new supporting lemmas.

Keywords and phrases: Split monotone Yosida variational inclusions; inverse strongly monotone operator;averaged operator; nonexpansive operator theorems.

AMS Subject Classication (2010): 47H05, 47H09, 47J25.

Connectivity of Modified Unit Graph of Some Commutative Rings

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Abstract

A graph is an instrument which is extensively utilized to model various problems in different fields. Up to date, many graphs have been developed to represent algebraic structures, particularly rings in order to study their properties. In this article, by focusing on commutative ring *R*, the modified unit graph associated with *R* and its complement are introduced. In addition, we prove that if the number of vertices of the modified unit graph be at least 2, then the graph is disconnected, while its complement graph is connected. Moreover, we will provide a counterexample to show the modified unit graph of ring *R* is connected and its complement graph is not connected, for some commutative rings without identity. Furthermore, it will be shown that there are some commutative rings with identity such as Boolean ring and the Cartesian product of Boolean rings in which their associated modified unit graphs are trivial.

Keywords: Commutative rings, Modified unit graph, Connectivity property

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Convergence and Stability of Perturbed Mann Iterative Algorithm with Errors for a System of Generalized Variational-Like Inclusion Problems

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Abstract

In this paper, we introduce a class of H(., .)-φ-η-accretive operators in a real q-uniformly smooth Banach spaces. We define the resolvent operator associated with H(., .)-φ-η-accretive operator and prove it is single-valued and Lipschitz continuous. Moreover, we propose a perturbed Mann iterative method with errors for approximating the solution of the system of generalized variational-like inclusion problems and discuss the convergence and stability of the iterative sequences generated by the algorithm. The results presented in this paper generalize and unify many known results in the literature.

Keywords: System of generalized variational-like inclusion problem, H(., .)-φ-η-accretive operator, q-uniformly smooth Banach spaces, resolvent operator technique, Perturbed Mann iterative method with errors, Convergence analysis, Stability analysis.

MSC-numbers: 47H10, 49J40

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Generalized Bernstein type operators on unbounded interval and some approximation properties

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ABSTRACT

In the present paper, we construct a new family of Bernstein type operators on infinite interval by using exponential function a^x. We study some approximation results for these new operators on the interval [0,∞).

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Convergence of Generalized Lupas-Durrmeyer~Operators

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Abstract

The main aim of this paper is to establish summation-integral type generalized Lupa\c{s} operators with weights of Beta basis functions which depends on μ having some properties. Primarily, for these new operators, we calculate moments and central moments, weighted approximation is discussed. Further, Voronovskaya type asymptotic theorem is proved. Finally, quantitative estimates for the local approximation is taken into consideration.

Keywords: generalized Lupa\c{s} operators; Beta function; Korovkin's type theorem; convergence theorems; Voronovskaya type theorem.

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Characterizations of Lie-type derivations of triangular algebras with local actions

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Abstract

Let N be the set of nonnegative integers and A be a (n-1)-torsion free triangular algebra over a commutative ring R. In the present paper, under some mild assumptions on A, it is prove that if $\delta : A \rightarrow A$ is an R-linear mapping satisfying $\delta(p_n(X_1, X_2, ..., X_n)) = \sum_{i=0}^{n} p_n(X_1, X_2, \ldots, X_{i-1}, \delta(X_i), X_{i+1}, \ldots, X_n)$ for all $X_1, X_2, ..., X_n \in A$ with $X_1X_2=0$ (resp. $X_1X_2=P$, where P is a nontrivial idempotent of A), then $\delta = d+\tau$; where $d : A \rightarrow A$ is a derivation and $\tau : A \rightarrow Z(A)$ (where Z(A) is the center of A) is an R-linear map vanishing at every (n-1)-th commutator $p_n(X_1, X_2, ..., X_n)$ with $X_1X_2=0$ (resp. $X_1X_2=P$).

Keywords: Lie derivation, Lie n-derivation, Triangular algebra.

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A new Approach to Fuzzy Decision-Making Problems under Probabilistic Interval Valued Hesitant Fuzzy Information

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Abstract

The information of aggregation operators is playing very important role in the decision support systems. Therefore, the aim of this paper is to work on the probabilistic interval-valued hesitant fuzzy aggregation operators, this paper examines the novel multi-attribute group decision-making (MAGDM) method to tackle the complete loss of knowledge in a hesitant fuzzy data setting. First, the concept of probabilistic interval-valued hesitant fuzzy set would be added, and some new probabilistic interval-valued hesitated fuzzy operators will be described using sine trigonometric q-rung orthopair and also notable work of sine trigonometry maintains the periodicity and symmetry of the origin in nature and thus satisfies the inclinations of decision-makers over the parameters of multi-stage. Secondly, based on these operations, the q-rung orthopair interval-valued probabilistic hesitant fuzzy ordered weighted averaging (q-ROIVPHFOWA) operator and the q-rung orthopair interval-valued probabilistic hesitant fuzzy ordered weighted geometric (q-ROIVPHFOWG) operator are proposed and their desirable properties will be discussed. We also analyze their common types and examine the relation between the suggested operators. Finally, a new probabilistic interval-valued uncertain fuzzy MAGDM model is developed and the viability and efficacy of the proposed model is confirmed by an example of acceptable hydraulic excavator.

Keywords: Hesitant Fuzzy Set, Probabilistic Hesitant Fuzzy Set, Decision Making Problems, Probabilistic Interval Valued Hesitant Fuzzy Aggregation Operators,
Acknowledgement: The author would like to thank DSR (UQU) for supporting this work under grant code 19-SCI-1-01-0055.

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A note on Delannoy and Schröder numbers

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Abstract

In this work, inspiring the combinatorial respects, we focus on Delannoy and Schröder numbers in order to present several formulas, which cover some conclusions on this topic.

Keywords: Delannoy number, Schröder number, Identity.

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Some Properties of Rough Statistical Convergence in 2-Normed Spaces

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Abstract

In this study, we introduce the concepts of rough statistical cluster point and rough statistical limit point of a sequence in 2-normed space and investigate some properties of these concepts.

Keywords: Rough convergence, 2-normed space, Statistical convergence, Rough cluster point, Rough limit point.

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Computing Some Eccentricity-Based Topological Indices of Line graphs of Dutch Windmill Graphs

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Abstract

Graph theory is very important area for mathematics, computer science, chemistry, and so on. There is a lot of application in daily life, especially it is used in chemistry. A chemical graph is a graph such that each vertex represents an atom of the molecule, and represents covalent bonds between atoms by edges of the corresponding vertices. Research on the topological indices has been intensively rising recently. Topological indices are the molecular descriptors that describes the structures of chemical compounds. The Dutch windmill graphs denoted by D_m^n that represents bidentate ligands in Chemistry. In this study, two eccentricity based topologicel indices namely the eccentric connectivity index $\xi^c(G)$ and the modified eccentric connectivity index $\xi_c(G)$ are computed for the line graphs of Dutch windmill graphs D_m^n. Then, the three-dimensional graphics of $\xi^c(L(D_m^n))$ and $\xi_c(L(D_m^n))$ are plotted with the help of space cartesian coordinate system.

Keywords: Graph theory, Distance, Eccentricity, Topological indices, Dutch windmill graphs, Line graphs.

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An Approach to Clebsch System by a Hirota Discretization

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Abstract

Apart from the cases that are the Euler, Lagrange, and Kowalewski tops and the Steklov system, the Clebsch system is also a rare system that can be integrated in Liouville sense and defines the equations of motion in the rigid body having being discovered by Clebsch. When the equations began to be studied, they were studied in a form of the Euler-Poisson equations of a rigid body. By the time, it has become necessary to define the Clebsch system as a parametric family of Hamilton vector fields on the dual of the Lie algebra of the group of motions of the Euclidean space E_3.

The equations of motion of a rigid body in an ideal fluid is given by the following system:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{x} \times \dfrac{\partial H}{\partial \boldsymbol{p}} \\ \dot{\mathbf{p}} = \mathbf{x} \times \dfrac{\partial H}{\partial \boldsymbol{x}} + \boldsymbol{p} \times \dfrac{\partial H}{\partial \boldsymbol{p}} \end{cases}$$

with $H(p,x)$ being a quadratic form in $p = (p_1, p_2, p_3) \in \mathbb{R}^3$ and $x = (x_1, x_2, x_3) \in \mathbb{R}^3$; here $\times$ denotes the vector product in $\mathbb{R}^3$. This system is Hamiltonian with the Hamilton function $H(p,x)$, with respect to the Poisson bracket $\{p_i, p_j\} = p_k, \{p_i, x_j\} = p_k$ where (i,j,k) is a cyclic permutation of (1, 2, 3).

Applying bilinear method and using the gauge invariance and the time reversibility of the equations, we get gauge-invariant bilinear difference equations. Finally, we derive the explicit discrete system by considering Hirota bilinear transformation method and present sufficient number of the discrete conserved quantities for integrability. Application of the Hirota type discretization of the Clebsch system leads to the discovery of four functionally independent integrals of motion of this discrete-time system, which turn out to be much more complicated than the integrals of the continuous-time system.

Keywords: Discretization, rigid body, bilinear method, gauge invariance.

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An Alternative Approach to Field Theory with Hypercomplex Numbers

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Abstract

Quaternion algebra [1, 2] is used to reformulate of many subfields of physics such as classical mechanics, electromagnetism, linear gravity and plasma physics in the different ways. In this study, both electromagnetism and linear gravity [3, 4] have been combined as the new perspective including dual-complex form of quaternions for the first time. By this manner, the single and basic notations have been established for the field strengths, source and potential equalities. Dual structures [5], which are significant definitions for screw movement in mechanics [6, 7], are presented again in the different physical systems. It is understood that the quaternionic descriptions are still valid and popular concepts for generalizing field equations in an alternative form.

Keywords: Algebraic structures, Dual number, Electromagnetism, Linear gravitation.

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On the strong solvability of the nonlinear parabolic equations

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Abstract

In this abstract, we annownce a strong solvability result for a class of nonlinear parabolic equations with principial part is heat operator and the rifht part is a given function which depend on the set of time and spatial variales, also depend on the solution and its spatial gradient. For such equation it has been proved a strong solvability result in the proper parabolic Sobolev space for the initial-boundary value problem. It is found a sufficient condition for the growth order of the right hand side in dependence of the gradient of the solution. The growth order contains a function coefficient, which also is a summable function. Using the Pohojaev's approaches from [1] it is established the proper result for the parabolic equations. The growth condition for the right hand side is a generalization of well-known Bernshtain's condition for elliptic equations. The domain is a sylindrical domain with its base is a smooth n-dimensional domain.

Keywords: parabolic equation, strong solution, growth condition, a priory estimate, heat operator, parabolic Sobolev space, smooth domain.

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Approximation by a Class of q-Beta Operators of the Second Kind Via the Dunkl-Type Generalization on Weighted Spaces

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Abstract

The aim of the present article is to study the approximation and other related properties of a class of q-Sz´asz–Beta type operators of the second kind. In this context, we construct the class of q-Sz´asz–Beta type operators of the second kind, which are generated by means of the exponential functions of the basic (or q-) calculus via the Dunkl-type generalization. In order to get a uniform convergence on weighted spaces, we obtain Korovkin-type approximation theorems involving local approximations and weighted approximations, the rate of convergence in terms of the classical, the second order and the weighted moduli of continuity, as well as a set of direct theorems. Relevant connection of the results presented in this article with those in earlier works is also indicated.

Keywords and phrases:Basic (or q-) calculus; Basic (or q-) integers; Basic (or q-) Beta functions; Basic (or q-) exponential functions; Dunkl's analogue; Generalized exponential functions;Sz´asz operator; Modulus of continuity; Peetre's K-functional; Weighted modulus of continuity;Korovkin-type approximation theorems.

AMS Subject Classification (2010): Primary 41A25; 41A36; Secondary 33C45.

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An interpolation inequality for weight cases

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Abstract

In this abstract, we propose an interpolation inequality with three weights, which estimates a weighted Lebesgue norm of the function through the multiplicate of other weighted Lebesgue norms of function and its derivative of the next type

$$\|f\|_{q,v} \le C_0 A^{1/q} \|f\|_{m,\omega_1}^{1/q'} \|Df\|_{p,\omega}^{1/q}, \tag{1}$$

where $Df = \left(\partial_{x_1} f, \partial_{x_2} f, \ldots, \partial_{x_n} f\right)$ The following main result is asserted by this abstract

Theorem. *Let* $m > 0, p \ge 1, q \ge \max\left(p, m(p-1)/p\right)$ *and* $D \subseteq \Re^n$ *be a domain (may be unbounded). Let the positive measurable functions* v, ω_1 *are of* A_∞ *-Muckenhoupt's class and* $\sigma = \omega^{1-p'} \in L^{1,loc}$ *. Then for the inequality (1) to hold for any function* $f \in Lip_0(\Omega)$ *it suffices that the Frostman's type condition*

$$|Q|^{1/n-1} v(Q) \sigma(Q)^{1/p'} \le A \omega_1(Q)^{(q-1)/m}$$

all over the balls $Q \in \Im$, $\Im = \{Q = Q(x,r) : x \in \Omega, 0 < r < d_\Omega\}$ *to be fulfilled, where* C_0 is a positive constant depending only on n, p, q and A_∞ -constants of the functions v, ω_1 .

For the subject we refer e.g. the book [1, 2]. Such inequalities find a usefull application in study of the regularity properties of the degenerate parabolic equations.

Keywords: regularity of solutions, interpolation inequality, embedding results, Harnack's inequality, Harnack's inequality, fundamental solutions, weak solutions, parabolic equations

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Blow up of Solutions for the p-Laplacian Wave Equation with Logarithmic Nonlinearity

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Abstract

The main goal of this paper is to study the blow up solutions for the p-Laplacian wave equation with logarithmic nonlinearity. The logarithmic nonlinearity arises in a lot of different areas of physics such as inflation cosmology, supersymmetric field theories, quantum mechanics and nuclear physics [1, 2]. By the motivation of this work, some authors studied the different mathematical bahaviour of different problems with logarithmic source term [3, 4]

Keywords: Blow up, p-Laplacian equation, logarithmic nonlinearity.

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Existence and Nonexistence for a nonlinear Viscoelastic Kirchhoff-type Equation with Logarithmic Nonlinearity

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Abstract

The equation with the logarithmic source term is encountered many branches of physics. It is well known, kind of logarithmic nonlinearity appears naturally in supersymmetric field theories and in inflation cosmology [1, 3]. By the motivation of this work, some authors studied the different mathematical bahaviour of different problems with logarithmic source term [2, 4]. In this paper, we investigate the initial boundary value problem of nonlinear viscoelastic Kirchhoff-type equation with logarithmic source term. We prove existence of solution and nonexistence of solutions.

Keywords: Existence, nonexistence, logarithmic nonlinearity.

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On ρ −Statistical Convergence of Sequences of Sets

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Abstract

Wijsman $\rho -$ statistical convergence and Wijsman strongly $\rho -$ statistical convergence are introduced in this work. Also, the relationships between these concepts are given.

Keywords: Statistical convergence, Cesàro summability, Strongly p-Cesàro summability, Wijsman convergence.

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Numerical Solutions of Linear Fractional Differential Equations by Genocchi Polynomials

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Abstract

Fractional differential equations (FDEs) are indispensable for many engineering disciplines thanks to their powerful modeling capacity. Since most of the FDEs do not have analytical solutions, it is essential to investigate the numerical solution methods for those FDEs. In this paper a numerical solution method for the fractional-order differential equations using Genocchi polynomials is proposed. Operational matrices for integer and fractional-order derivatives are obtained employing Genocchi polynomials. By using those operational matrices, the fractional differential equation is converted to an algebraic equation in vector-matrix form. By calculating the algebraic equation for a few collocation points and also incorporating the initial conditions, a system of algebraic equations is obtained. The solution of those algebraic equations provides the coefficient vector c, which, in turn yields the numerical solution of the fractional-order differential equation. Numerical example results demonstrate that the numerical solution is a very accurate approximation to the FDE.

Keywords: Genocchi polynomials, Genocchi operational matrix of fractional derivatives, numerical solutions for fractional differential equations.

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On the Solvability Dirichlet Problem for the Laplace Equation with the Boundary Value in Grand-Lebesgue Space

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Abstract

In this paper the weighted grand space of harmonic within the unit circle of functions $h_w^{p),\theta}$ is defined and the solvability of the Dirichlet problem for the Laplace equation in this space is considered. Using the boundedness of the maximum operator in the weighted grand-Lebesgue space, the solvability of the Dirichlet problem for the Laplace equation with a boundary value from the grand-Lebesgue weight space is proved.

Keywords: Laplace equation, Dirichlet problem, weighted grand-Lebesgue space

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New Answers to the Rhoades' Open Problem and the Fixed-Circle Problem

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Abstract

In this talk, we give some solutions to the Rhoades' open problem and the fixed-circle problem on metric spaces. To do this, we inspire from the Meir-Keeler type, Ciric type and Caristi type fixed-point theorems.

Keywords: Fixed point, fixed circle, fixed disc, metric space.

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Numerical Analysis of Transient Turbulent Flow in Domical Roofed Structures

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Abstract

Factory buildings, shopping centers, stations, theaters etc are typical examples of domical roofed structures where two-dimensional analysis is valid. In this study transient buoyancy driven natural convection heat transfer and the turbulent air flow in domical roofed enclosures is numerically investigated. The Rayleigh number range accepted in two-dimensional analyzes is $10^8 \leq Ra \leq 10^{13}$. The aspect ratio, H/L, is defined as the ratio of the lateral face height to the base face length of the structure and it is considered equal to 1. The aspect ratio, h/H, is defined as the ratio of the dome height to the lateral face height of the structure and it is accepted equal to 0.5 in this study. Two lateral surfaces of the enclosure are heated while the domical surface is cooled. The bottom surface is considered adiabatic. The hot and cold surfaces are considered isothermal. The related governing equations are solved using Ansys Fluent 2020 R1 software. The RNG *k-ε* turbulence model and the Boussineq approximation modeling the buoyancy flow are used. The streamlines and isotherms in the enclosure are presented for all the Ra numbers studied. The mean Nusselt number is evaluated over the isothermal cold wall, and the results are compared with respect to the Rayleigh numbers studied.

Keywords: Buoyancy, Domical, Natural convection, Turbulent flow.

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The Succesive Approximations Method for Solving Non-Newtonian Fredholm Integral Equations of the Second Kind

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Abstract

In this study, the Fredholm integral equations are defined in the sense of non-Newtonian calculus. The successive approximations method is applied to solve the non-Newtonian linear Fredholm integral equations of the second kind and the conditions are investigated to uniqueness of the solution.

Keywords: Non-Newtonian calculus, non-Newtonian Fredholm integral equations, successive approximations method

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On Ideal Invariant Convergence of Double Sequences in Regularly Sense

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Abstract

In this paper, we defined concepts of $r(\sigma,\sigma_2)$-convergence, $r[\sigma,\sigma_2]$-convergence, $r[\sigma,\sigma_2]_p$-convergence, $(r[\sigma,\sigma_2]_p - convergent)$, $r\left(I_{\sigma,}I_2^{\sigma}\right)$-convergence of double sequences . Also we research the relationships among them.

Keywords: Double sequences, Regularly ideal convergence, invariant convergence.

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On Lacunary Ideal Invariant Convergence of Set Sequences in Wijsman Sense

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Abstract

In this paper, we defined concepts of $\mathcal{I}_{\sigma\theta}^{W}$-convergence and $[WN_{\sigma\theta}]_p$-convergence of sequences of sets. Also, we research the relationships among $\mathcal{I}_{\sigma\theta}^{W}$-convergence and $[WN_{\sigma\theta}]$- convergence.

Keywords: Lacunary convergence, ideal convergence, invariant convergence, set sequences.

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Zero Divisor Graph of Ring of Matrices Over Some Finite Fields And Its Applications

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Abstract

The study of graph theory was introduced and widely researched since many practical problems can be represented by graphs. A directed simple graph is said to be a zero-divisor graph, if two different elements of the ring are the vertices of the graph denoted as x and y, such that x and y are adjacent with each other if and only if $xy = 0$. In this study, we focus on the zero-divisor graph of ring of matrices over finite fields. First, the zero divisors of the ring of matrices over some finite fields are found. Next, the zero-divisor graph are constructed based on the zero divisors obtained from the ring of matrices over some finite fields. Then, some properties of the graph, such as chromatic number, edge-chromatic number, clique number, girth and the diameter are also found in this study.

Keywords: Zero divisors, zero-divisor graphs, ring of matrices, fields.

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A Comprehensive Survey of Dual Generalized Complex Fibonacci Numbers

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Abstract

The aim of this paper is to develop dual-generalized complex Fibonacci numbers. Fibonacci sequence is defined by $F_{n+1} = F_n + F_{n-1}, n \geq 1$ where $F_0 = 0, F_1 = 1$ and it is special type of Horadam sequence. Moreover, the general recurrence relations of dual-generalized complex Fibonacci and Lucas numbers are obtained. Also, Binet's formulas, D'Ocagne's, Cassini's and Catalan's identities are calculated for these type of numbers.

Keywords: Dual-generalized complex number, Fibonacci number.

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A perturbation procedure for a multi-component beam with high contrast properties in case of lowest vibration modes

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Abstract

In this work, free vibration modes of a multi-component beam with the arbitrary number of components composed of strongly varying material properties in the case of free boundary condition are investigated. It is observed that softer components of the beam asymptotically contribute to an almost rigid-body motion of the stiffer parts and give rise to two nonzero eigenfrequencies contrary to a single beam with free end conditions. An asymptotic procedure is employed to derive the eigenfrequencies as well as the eigenforms revealing that only under certain conditions on the ratios of material parameters low-frequency, non-rigid body motions are also possible. Numerical illustrations are presented to confirm that the obtained asymptotic frequencies agree well with the exact frequency in the lowest frequency range. Comparisons of asymptotic and exact are also presented and a remarkable agreement is observed for high-contrast beam components.

Keywords: Multi-component beam, contrast materials, low frequency vibration, perturbation procedure.

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Analytical solution methods for the multidimensional partial differential equations of the hyperbolic type

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Abstract

In this we study on the analytical solution methods for the multidimensional partial differential equations (pdes). Some of the well known analytical methods are applied to solve the multidimensional hyperbolic partial differential equations.

Keywords: Hyperbolic pdes, Fourier series, Fourier transform.

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The Topology of δ_w-open Sets

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Abstract

Weak and strong forms of the notion of open set play a very important role in general topology and have been studied by many authors. Some of them such as the notions of δ-open [2], θ-open [2] and regular open [1] sets are stronger notions than the notion of open set. However, the notion of an *w*-open [3] set is weaker than the notion of an open set. The relation between the notions which are mentioned above is as follows:

θ-open → δ-open → open → *w*-open
↑ (from Regular open to δ-open)
Regular open

Recently, Samer Al Ghour et. al. introduced θ_w-open [4] sets as a new class of sets utilizing the θ_w-closure operator as a new topological operator. The authors have shown that the class of θ_w-open sets lies strictly between the class of θ-open sets and the class of open sets. In this study, we introduce the notion of the δ_w-open set via δ_w-closure operator as a new topological operator. We give some fundamental properties of the notion of δ_w-open set. Moreover, we show that the class of δ_w-open sets lies strictly between the class of δ-open sets and the class of open sets. Also, we prove that the notion of the δ_w-open set is weaker than the notion of the θ_w-open set. Finally, we show that the class of all δ_w-open sets in a topological space forms a topology which is finer than the old one.

Keywords: δ_w-closure operator, δ_w-closed, δ_w-open, θ_w-open.

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Some Results About Invariant Subspaces

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Abstract

In this work, it is obtained some results about invariant subspaces of weakly compact friendly operators.

Keywords: Invariant subspaces, Banach lattices, Weakly compact-friendly operators,

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A Study on Tangent Bundle of the Hypersurface

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Abstract

In this work, the complete lift of Weyl connection on tangent bundle of the hypersurface is determined. Under some conditions, we find certain results on totally umbilical and geodesic with respect to complete lift of Weyl connection.

Keywords: Hypersurface, weyl connection, tangent bundle, complete lift.

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Convergence Of The Biorthogonal Expansion In Root Functions Of An Odd Order Differential Operator

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Abstract

We study the absolute and uniform convergence of biorthogonal expansions of functions of the class $W_2^1(0,1)$ in the root functions of ordinary differential operator

$$Lu = u^{(2m+1)} + P_1(x)u^{(2m)} + ... + P_{2m+1}(x)u, \quad m \in N,$$

with coefficients $P_l(x) \in W_1^{(2m+1-l)}(0,1)$.

Sufficient conditions of absolute and uniform convergence a obtained and rate of uniform convergence of these biorthogonal expansions on the interval [0,1] is found.

Keywords: biortoqonal expansion, absolute and uniform convergence, root functions

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Bacterial Population Models with Caputo Katugampola Fractional Derivative

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Abstract

In this work, some bacterial population models are considered with Caputo Katugampola fractional derivative, which has the characteristics of Caputo and Hadamard fractional derivatives. Results obtained for bacterial population models are compared under different fractional orders and different parameter values by means of some illustrations.

Keywords: Fractional derivatives, Caputo Katugampola fractional derivative, Bacterial population models.

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Optimality Conditions in One Stochastic Control Problem

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Abstract

Considered on the interval $T = [t_0, t_1]$ a linear stochastic controlled system in the form:

$$I(u) = c'x(t_1) \to min, c \in R^n = const,$$
$$dx(t) = \left[A(t)x(t) + f\left(t, u(t)\right)\right]dt + B(t)x(t)dw(t), t \in T,$$
$$x(t_0) = x_0.$$

Here $A(t), B(t) -$ are given $n \times n -$ matrices, $f(t, u) -$ is a given n-dimensional vector - function, $w(t) -$ is $n -$ dimensional standard Wiener process, control $u \in R^r$.

The necessary and sufficient optimality condition in the form Pontryagin maximum principle is proved.

In the case of convexity of the quality functional, a sufficient optimality condition is obtained.

Keywords: Ito equations, stochastic system, optimal control.

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COPRIME INTEGER ENCRYPTION ALGORITHM UPON EULER'S TOTIENT FUNCTION'S UNSOLVED PROBLEMS

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Abstract

Euler's totient function is a function that, for the natural number *n*, gives the number of coprime integers to *n*. However, there is no study of what these numbers are. The prime goal of this study is to address this problem. Based on groups, isomorphisms, and isomorphic groups; a method has been developed through which all of these numbers can be found using modular arithmetic. Based on these numbers, an encryption algorithm has been developed. While developing this algorithm, cartesian multiplicative groups isomorphic to the Z_n group were used. The most important feature of the encryption algorithm is that two or more layers of encryption can be done. For example, the backing up of the social media accounts' conversations or data by the relevant social media company or software is prevented. This algorithm and its software will help communication and transfer of data especially between strategically important institutions in our country. For example, it prevents the intermediary institutions from accessing the relevant information as they relay the decisions taken by the General Staff to other institutions; as it is encrypted over and over continuously and differently while institutions relay information or data. The importance of the topic is better understood considering the interests of our country especially in the present.

Keywords: Euler's totient function, groups, isomorphism, encryption algorithm, abstract algebra

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Textile Pattern Detection with Line, Circle, Corner and Co-occurrence Matrix Features

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Abstract

In this study, the hybrid features as line, circle, corner, and co-occurrence matrix [1] have been presented for textile pattern detection [2, 3, 4] due to the very rapid increase of textile images. Today, it is necessary to classify textile images automatically since the classification by hand is a cumbersome and time-consuming operation. The detection systems of textile patterns are first trained with the train set. Then, the success of the systems is determined by the test set. In the pre-processing stage and feature extraction, color or indexed images are converted to a grayscale image, and edge detection operation is applied to the grayscale. Line, circle, corner, and gray-level co-occurrence matrix features are obtained for training and testing of the textile images which consist of seven textile pattern categories. With the training data set, the models for each class are generated with Support Vector Machine [5], Linear Logistic Regression, and the C4.5 decision tree algorithm [6]. These trained models detect which class the image in the test set belongs to. To measure the accomplishment of the systems, f-score and accuracy metrics are used with the k-fold-cross-validation technique. The results of the systems according to the methods have been evaluated and compared.

Keywords: Textile patterns, GLCM, Corner detection, Line detection, Circle detection, Support Vector Machine, Linear Logistic Regression, C4.5.

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Syllable and Word-Based Speech Recognition Using Multi-Layer Perceptron

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Abstract

In this study, syllable and word-based Turkish speech recognition systems have been designed and implemented with Multi-Layer Perceptron. The developed recognition systems consist of five stages: pre-processing, feature extraction, training models, speech recognition, and post-processing. In the pre-processing stage, the operations as speech signal smoothing, windowing, and syllable end-point detection have been applied to the speech signals. In the feature extraction, the speech features as mel frequency cepstral coefficients, linear predictive coefficients, and parcor coefficients are extracted, and the feature vectors are generated for syllable and word utterances. All syllables and words in the middle scaled Turkish dictionary have been trained by Multi-Layer Perceptron. Then, the syllable and word models are obtained for this method. The recognized word is determined by the models in the recognition stage. For syllable based recognition, the recognized words have been constituted by the concatenation of the recognized syllables. In the post-processing stage, using Turkish syllable n-gram frequencies, the system accepts as the recognized word if the word is Turkish. According to Multi-Layer Perceptron, the syllable-based systems with mel frequency cepstral coefficients outperform the word-based systems.

Keywords: Syllable-based speech recognition, Word-based speech recognition, Multi-Layer Perceptron, mel frequency cepstral coefficient, Linear predictive coefficients, parcor coefficients.

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On Weighted Criterion For Hausdorff Operator in Lebesgue Spaces

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Abstract

We establish necessary and sufficient condition for the boundedness of Hausdorff operator, namely in the form of boundedness condition for the Hardy operator acting between weighted Lebesgue spaces.

Keywords: Weighted Lebesgue spaces, Hausdorff operator, monotone weight functions, one weight inequalities.

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Normal S-Iterative Algorithm for Solving General Variational Inclusions Involving Difference of Operators

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Abstract

In this presentation, we propose a normal S-iterative algorithm and analyze this algorithm for finding a zero of the difference of operators. We also discuss the convergence of this algortihm under some mild conditions in a Hilbert space. Our results may be considered as refinement and improvement of the some earlier results in the literature.

Keywords: Normal S-iterative algorithm, General variational inclusions, Convergence, Hilbert space.

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Weighted statistical convergence based on difference operator with associated approximation theorems

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Abstract

We discuss the notion of weighted statistical convergence based on difference operator and investigate its relationship with the notions of statistical convergence, weighted statistical convergence as well as other related notions. We establish the Korovkin-type approximation theorem and then construct an illustrative example by taking positive linear operators which proves that our approximation theorem is stronger than some existing approximation results in the literature. Moreover, we prove Voronovskaya-type approximation theorem with the help of our newly defined convergence method.

Applications of Soft intersection sets in Hypernear-rings

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Abstract

In this paper, we define soft intersection hypernear-rings and shows how a soft set effects on a hypernear-ring structure by means of intersection and insertion of sets. Further, we explore some properties using hypernear-ring theoretic concepts for soft sets.

Keywords: Hypernear-rings, Soft intersection sets, Soft intersection hyperideals.

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Double Bases from Generalized Faber Polynomials with Complex-valued Coefficients in Weighted Lebesgue Spaces with General Weight

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Abstract

In this work we consider the generalized Faber polynomials defined inside and outside a regular curve on the complex plane. Weighted Smirnov spaces are introduced and it is proved that the generalized Faber polynomials form a bases in these spaces provided that the weight function satisfies the Mockenhaupt condition on the curve. The basisness of the double system of generalized Faber polynomials with complex-valued coefficients in the weighted Lebesgue spaces is also studied.

Keywords: Faber polynomials, Smirnov classes, weight, basisness

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Invariants of the Z2 orbifolds of the Podleś two spheres

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Abstract

There are two Z2 orbifolds of the Podleś quantum two-sphere, one being the quantum two-disc Dq and other the quantum two-dimensional real projective space RPq2. In this article we calculate the Hochschild and cyclic homology and cohomology groups of these orbifolds and also the corresponding Chern–Connes indices.

Mathematics Subject Classification (2010). 19-xx; 17-xx.

Keywords: Podleś sphere, Chern–Connes index, homology, non-commutative spheres.

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The convolution for the Mehler-Fock transform revisited

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Abstract

In this paper, various weigted estimates of generalized Mehler-Fock translation and convolution operators are obtained in the Lebesgue spaces. We presented a solution of linear parabolic differential equation involving the generalized convolution operator.

Keywords: Mehler-Fock transform, Legendre function, Generalized convolution operator, Lebesgue space.

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Comparative study between certain encryption algorithms on BMP and JPEG images

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Abstract

In this article we carried out a comparative study between some encryption algorithms on BMP and JPEG images, we established a comparison between some types of encryption systems and our algorithm. We made the comparison with data implemented on the same Intel (R) computer equipped with a processor (Pentium 4) clocked at 1.8 MHz and under a RAM of 256 MB and a Windows XP operating system and ours implemented on Intel (R), Core (TM), i5-3340M, CPU @ 2.70GHz, 2.00GB RAM, Windows 10 operating system.

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Keywords: Image, Algorithm, Encryption, Comparative.

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On Liouville Theorem For Degenerated Parabolic Equations

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Abstract

In this abstract, if has been stated the Liouville's theorem for a weak solutions of degenerated parabolic equations

$$\frac{\partial}{\partial x_i}\left(a_{ij}(t,x)\frac{\partial u}{\partial x_i}\right)=0, \qquad x\in D$$

with the conditions:

i) $$c_1\omega(t,x)|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \le c_2|\xi|^2\omega(t,x)$$

ii) Here $\omega(t,x)$ is a positive a.e.in D function satisfying some summability and Muckenhoupt type condition all over the special cylinders with height depending on $\omega(t,x)$.

Theorem. Let $u(t,x)$ be a solution of (1) in the half space $t\le t_0$ such that,

$$|u(t,x)|\le M .$$

Then $u(t,x)\equiv 0.$

Keywords: Harnack's inequality, Moser iteration, parabolic equations, Holder continuity, Liouville theorem.

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Hyers-Ulam-Rassias stability of a boundary value problem with integral boundary conditions

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Abstract

In this work, by applying Diaz-Margolis's fixed point theorem, we study the Hyers-Ulam and Hyers-Ulam-Rassias stability of a second-order boundary value problems with integral boundary conditions.

Keywords: Hyers-Ulam stability, Hyers-Ulam-Rassias stability, boundary value problem, integral boundary conditions, fixed point.

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A Note on τ-quasi Ricci-Harmonic Metrics

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Abstract

In this work, τ-quasi Ricci-Harmonic metrics are examined on a warped product manifold M. We obtain a result on the domains of potential function and the mapping ϕ:M→N and find a rigidity result for the warped product manifold.

Keywords: τ-quasi Ricci-Harmonic metrics, harmonic Einstein, warped product

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On linear operators giving higher order approximation of functions in $L_{\sigma}^{p}(R^{+})$

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Abstract

Numerical investigations of different authors were devoted to convergence and convergence of singular integrals, approximation of functions by linear operators. Asymptotic value of approximation of function by linear operators were abtained.

Keywords: . High approximation order, multiply differentiable functions, bounded variation Laplace-Stieltjes transformation of the function,

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Approximation properties of λ-Bernstein-Kantorovich operators with shifted knots

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Abstract

In the present article, Kantorovich variant of λ-Bernstein operators with shifted knots are introduced. The advantage of using shifted knot is that one can do approximation on [0,1] as well as on its subinterval. In addition, it adds flexibility to operators for approximation. Some basic results for approximation as well as rate of convergence of the introduced operators are established. The r^{th} order generalization of the operator is also discussed. Further for comparisons, some graphics and error estimation tables are presented using MATLAB.

Keywords: λ-Bernstein operators, Kantorovich operators, Rate of convergence, Modulus of continuity.

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THE DIRICHLET PROBLEM FOR OF SEMILINEAR ELLIPTIC EQUATIONS OF THE SECOND ORDER

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Abstract

We consider in Ω the following Dirichlet problem:

$$\sum_{i,j=1}^{n} a_{ij}(x)u_{x_i x_j} + g(x,u) = f(x), x \in \Omega, \quad (1) \qquad u|_{\partial\Omega} = 0 \quad (2)$$

Here $a_{ij}(x), i,j = 1,2,..,n$ are bounded measurable functions satisfying the conditions
a) uniform ellipticity condition
b) Cordes's condition.

Here $\operatorname{esssup}_{x\in\Omega} \frac{\sum_{i,j=1}^{n} a_{ij}^2(x)}{(\sum_{i=1}^{n} a_{ii}(x))^2} \leq \frac{1}{n-1} - \delta$, $g(x,u): \Omega \times E_1 \to E_1$ is a Caratheodory function satisfying

c) $|g(x,u)| \leq b_0(x)\,|u|^q$, $b_0 \phi 0$, $b_0 \in L_s(\Omega)$, $s \phi 2$.

Theorem. Let $n \phi 4$, $s > 2, 1 \leq q < \frac{n(s-2)}{(n-4)s}$ and conditions (a)-(c) be satisfied, $\partial\Omega \in C^2$. Then there exists a sufficiently small positive constant $C_2 = C_2(n, \gamma, \delta, q, b_0, \Omega)$ such that problem (1)- (2) has at least one solution from $\dot{W}_2^2(\Omega)$ for any $f(x) \in L_2(\Omega)$ satisfying

$$\|f\| \leq C_1 (mes_n \Omega)^{-\left(\frac{n(s-2)-s(n-4)q}{2ns(q-1)}\right)}$$

For the proof we have applied the Schauder's fixed point theorem on a continuous into mappings of a convex set in Banach space.

$$\|f\|_{L_2(\Omega)} \leq C_7 (mes_n \Omega)^{-\left(\frac{n(s-2)-s(n-4)q}{2ns(q-1)}\right)},$$

Keywords: semilinear elliptic equation, Cordes's condition, Shauder's principle.

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Klein-Gordon equation in a symetric gauge field in a non-commutative complex space

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Abstract

In this work we obtain the exact solution for Landau problem and the Klein-Gordon oscillator in a symmetric gauge field in a non-commutative complex space, the corresponding exact of the energy spectrum is obtained. It is shown that the Landau problem and the Klein-Gordon os-
cillator in a symmetric gauge field, in a non-commutative complex space are similar behaviors the singal electron in the presence spin-orbit interaction in in a symmetric gauge field in commutative space. We shown that In the critical point, the landau levels are removed
degenerate and at this point, we get a particle that spins with no ongular momentum operator.

Keywords: Complex space, harmonic oscillator, noncommutative gauge theory.

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On Statistical Convergence of Measurable Functions in Probabilistic Normed Spaces

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Abstract

We study the concept of density of moduli with respect to μ-statistical convergence at a point for measurable functions in a measurable space. We also generalize the concept of μ-statistical convergence in a Probabilistic Normed space too.

Keywords:
μ-statistical convergence, f-statistical convergence, Probabilistic Normed Spaces,

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A modified weak Galerkin finite element method for thetime dependent convection diffusion reaction problems

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Abstract

In this paper, we propose a modified weak Galerkin finite element method for solving the time fractional convection diffusion reaction problems. The method is based on the so called the modified weak derivative on totally discontinuous weak function spaces. The key feature of this newly defined method is to replace the classical gradient operator by a modified weak gradient operator. We apply the backward finite difference method in time and the modified weak Galerkin finite element method in space on uniform mesh. The stability analyses are proved for both continuous and discrete time weak Galerkin finite element methods. The optimal order error estimates in L_2 and H^1 norms for both schemes are given. Finally, we give some numerical experiments to verify numerically the theoretical findings.

Keywords: Time dependent diffusion equations, modified weak Galerkin finite-element method, stability, convergence.

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Existence and Uniqueness of Solutions for the high-order Riezs-Caputo Fractional Boundary Value Problems with Impulsive

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Abstract

This paper concerns the existence and uniqueness of solutions for a class of high order fractional initial/boundary value problems of the Riesz-Caputo differential equations with impulsive. Sufficient conditions for the existence of solutions for the problem have been established.
Numerical examples are given to verify the results.

Keywords: Fractional functional differential equation Impulse Existence Uniqueness

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Contrabution to the Natural Hamiltonian Problem for Euclidean curves

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Abstract

In this work, we study one of the natural Hamiltonian problem generated by Frenet frame of a curve in three-dimensional Euclidean space. This variational problem occurs by derivative of the principal normal vector field of the curve. We derive the Euler-Lagrange equations related to the minimization of this natural Hamiltonian functional which is given depending on the boundary conditions. Then we find Killing vector fields along the critical curve in order to construct a cylindiral system. So, the critical curves are expressed by quadatures in cylindrical coordinates.

Keywords: Curvature, natural Hamiltonians, variational calculus, torsion.

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Pascal Type Distribution Series for a Subclass of Analytic Univalent Functions

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Abstract

The aim of the present paper is to develop some sufficient conditions for the Pascal type distribution series to be in the subclass $UTS^{*}(\gamma)$ of analytic univalent functions.

Keywords: Analytic univalent functions, distribution series

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Some Types of f-Biharmonic Legendre Curves in 3-Dimensional Normal Almost Paracontact Metric Manifolds

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Abstract

In this paper, we study non-null f-biharmonic Frenet and non-Frenet curves in three dimensional normal almost paracontact metric manifolds. We give necessary and sufficient conditions for Frenet and non-Frenet curves to be f-biharmonic under certain conditions.

Keywords: f-biharmonic curves, Legendre curves, normal almost paracontact metric manifolds.

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Characterization of the critical diameter for the graphene wrinkle model

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Abstract

In the present work, we study the wrinkling of Graphene put on a silica substrate decorated with silica nano-particles with diameter d. Indeed, Yamamoto showed in [1] that the profile of the wrinkle is the minimum of the total energy associated to the deformation. In this work, we show the existence of a critical diameter d^* beyond which the existence of such a minimum is insured in a suitable space K that we introduce using the super-solutions of the problem and the first eigenfunction of the p-laplacian. Furthermore, this minimum verifies the associated Euler-Lagrange equation which is a quasilinear elliptic equation with singular nonlinearity containing the p-laplacian and a Dirac mass at the origin. Last but not least, numerical investigations are carried out to determine the profile of the graphene wrinkle between two nanoparticles of diameter d for $d >= d^*$.

Keywords: graphene, wrinkling, diameter, Dirac, eigenfunction, p-laplacian, quasilinear, elliptic, existence, minimum.

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Common Fixed Point Theorems for Enriched Kannan Semigroups in Banach Spaces

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Abstract

In this work, we introduce a new semigroups of an enriched Kannan mapping, namely, an enriched Kannan semigroups. Furthermore, we prove some weak and strong convergence results for enriched Kannan semigroups to approximate common fixed points using Mann iterative process in uniformly convex Banach spaces. To support our results we present some illustrative examples.

Keywords: Common fixed point, enriched Kannan mapping, semigroup, strong convergence, weak convergence, Mann iterative process.

On the Exact Solutions of a Nonlinear Time Fractional Equation via IBSEFM

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Abstract

Fractional differential equations arise from the most part from the mathematical model of physical phenomena such as viscoelasticity, physics, control theory of dynamical systems which are usually modeled with nonlinear differential equations. Several computational techniques for the solutions of these equations have been developed.

In this study, we implement the Improved Bernoulli Sub-Equation Function Method (IBSEFM) to construct the exact solutions of conformable time fractional nonlinear partial differential equation. The results show that IBSEFM is an effective mathematical tool to solve nonlinear conformable time-fractional equations arising in mathematical physics.

Keywords: Exact solutions, Conformable Time Fractional Derivative, Improved Bernoulli Sub-Equation Function Method.

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On Holder regularity of the degenerated parabolic equations

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Abstract

In this abstract it has been studied the Holder continuity of the weak solutions of the degenerate parabolic equations

$$\frac{\partial}{\partial x_j}\left(a_{ij}(t,x)\frac{\partial u}{\partial x_i}\right)-\frac{\partial u}{\partial t}=0 \qquad (1)$$

with uniform degeneratacy condition

$$\frac{1}{C}(|x|^{\alpha}+t)|\xi|^2 \le a_{ij}(t,x)\xi_i\xi_j \le C(|x|^{\alpha}+t)|\xi|^2 \quad (2)$$

for $C>0$, $\forall\xi\in\Re^n,(t,x)\in D$, and D be a bounded domain in half-space $\{t<t_0\}$. It has been found a sufficient condition on the range of $\alpha\in\Re$ for the weak solutions of (1) to be Holder continues in the origin. For the subject we refer e.g. the works [1, 2]

Keywords: regularity of solutions, Holder regularity, Harnack's inequality, fundamental solutions, weak solutions, qualitative properties of solutions, a prior estimates.

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On the Mathematical Expectation of the Reinsurance Surplus Process

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Abstract

Reinsurance is one of the major risk and capital management tools available to primary insurance companies. Reinsurance is insurance for insurers. Insurers buy reinsurance for risks they cannot or do not wish to retain fully themselves. We call the insurer's surplus process as reinsurance surplus process when the insurer effects reinsurance.

Basically, there are some types of reinsurance contracts: proportional reinsurance, excess of loss reinsurance and excess stop loss reinsurance. If the insurer effects reinsurance, then the amount of claim paid by insurer is given by a function h in each type of reinsurance, so, if the amount of claim is x, then the insurer pays the amount of $h(x)$: $0 \leq h(x) \leq x$ (see, for example, [2-5]).

We consider reinsurance surplus process and derive asymptotic expansion for the mathematical expectation of this process.

Keywords: Reinsurance, surplus process, mathematical expectation.

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Lupas Blending Functions with shifted knots and q-Bezier Curves

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Abstract

In this paper, blending functions of Lupas q-Bernstein operators with shifted knots for constructing q-Bezier curves and surfaces are introduced. Nature of degree elevation and degree reduction for Lupas q-Bezier Bernstein functions with shifted knots for $t \in [\frac{a}{[\mu]_{q+b}}, \frac{[\mu]_q+a}{[\mu]_q+b}]$ has been studied. For the parameter a = b = 0; we get Lupas q-Bezier curves defined on [0; 1]. It has been shown that Lupas q-Bernstein functions with shifted knots are tangent to fore-and-aft of its polygon at end points. A de Casteljau algorithm to compute Bernstein Bezier curves and surfaces with shifted knots are presented. The new curves have some properties similar to q-Bezier curves.

Keywords: Degree elevation; de casteljau type algorithm; Bezier curve; Lupas q- Bernstein operators with shifted knots.

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Stability of Two Generalized Set-Valued Functional Equations

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Abstract

The present work aims to establish the Hyers–Ulam–Rassias stability of the following generalized set–valued functional equations

$$F\left(\frac{x+y}{2}\oplus(\alpha-1)z\right)\oplus F\left(\frac{x+z}{2}\oplus(\alpha-1)y\right)\oplus F\left(\frac{y+z}{2}\oplus(\alpha-1)x\right)=\alpha\left(F(x)\oplus F(y)\oplus F(z)\right),$$

$$F(\beta x+y)\oplus F(\beta x-y)=F(x+y)\oplus F(x-y)\oplus 2\left(\beta^2-1\right)F(x),$$

on vector spaces, where $\alpha\geq 2$ and $\beta\notin\{-1,0,1\}$ are fixed integers. These two equations are respectively related to Cauchy–Jensen type and quadratic type set–valued functional equations.

Keywords: Set–valued functional equation, set–valued functional inequality, stability.

On basicity of the system of exponents and trigonometric systems in the weighted grand-Lebesgue spaces

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Abstract

In the paper we study the basicity of the system of exponents and trigonometric system in the weighted grand-Lebesgue spaces $L_{p),\rho}$. Based on shift operator, we define the subspace $G_{p),\rho}(0,1)$ of the space $L_{p),\rho}(0,1)$, where continuous functions are dense, and study some properties of the functions belonging to this space. We establish the basicity of exponential system $\left\{e^{i\pi nt}\right\}_{n\in Z}$ for $G_{p),\rho}(0,1)$ and the basicity of trigonometric systems $\{\sin nt\}_{n\geq 1}$ and $\{\cos \pi nt\}_{n\geq 0}$ for $G_{p),\rho}(0,1)$, $1<p<+\infty$, when weight function ρ satisfy the Muckenhoupt condition.

Keywords: grand-Lebesgue space, basicity, exponential system, Muckenhoupt condition.

Let ρ be a some weight function and $L_{p),\rho}(0,1)$, $1<p<+\infty$, be a weighted grand-Lebesgue space with norm $\|f\|_{p),\rho}=\|\rho f\|_{p)}$, where $\|\cdot\|_{p)}$ is a norm in $L_{p)}(0,1)$. Let $G^{p)}(0,1)$ a closure of the set $C_0^\infty[0,1]$ in $L_{p)}(0,1)$ and $G_{p),\rho}(0,1)=\left\{f:\rho f\in G_{p)}(0,1)\right\}$.

The following statements are valid.

Theorem 1. *Let the weight ρ satisfy the Muckenhoupt condition. Then the system of exponents $\left\{e^{i\pi nt}\right\}_{n\in Z}$ forms a basis in the space $G_{p),\rho}(-1,1)$, $1<p<+\infty$.*

Theorem 2. *Let the weight ρ satisfy the Muckenhoupt condition. Then the system of sines $\{\sin nt\}_{n\geq 1}$ and cosines $\{\cos \pi nt\}_{n\geq 0}$ forms bases in the space $G_{p),\rho}(0,1)$, $1<p<+\infty$.*

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On basicity of eigenfunctions of one discontinuous spectral problem in weighted grand-Lebesgue spaces

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Abstract

In the paper the subspace $G_{p),\rho}(0,1)$ of weighted grand-Lebesgue space $L_{p),\rho}(0,1)$ generated by shift operator is considered. Teorems on the basicity of the eigen and associated functions of some discontinuous spectral problem for a second order differential equation with spectral parameter in boundary condition in weighted spaces $G_{p),\rho}(0,1)\oplus C$ and $G_{p),\rho}(0,1)$ with a general weight function $\rho(\cdot)$ satisfying the Muckenhoupt condition are proved.

Keywords: weighted grand-Lebesgue space, discontinuous spectral problem, basicity, Muckenhoupt condition.

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A Discussion on Stochastic Behaviour of Some Stiff Equations

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Abstract

In this study, deterministic and stochastic behaviour of some model equations have been investigated. To achieve this, various Taylor based techniques have been used in a comparative way. At the first stage of this research, very satisfactory results have been produced. The responses of the model equations have been discussed illustratively. In solving the problems of interest, computer codes have been produced in MATLAB.

Keywords: Ito stochastic ordinary differential equations, Ito-Taylor expansion.

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Investigation of a Non-Linear Cramér–Lundberg Risk Model

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Abstract

In this study, a special case of non-linear Cramér-Lundberg risk model is considered and investigated. In literature, a linear form of this model is usually defined as follows:

$$\boldsymbol{U(t) = u + ct - S(t)} \qquad (1)$$

The risk process $U(t)$ in Eq.(1) expresses an amount of capital of an insurance company at a given time t, the constant u is initial capital of the company, c – the premium rate, $S(t) = \sum_{i=1}^{N(t)} X_i$ is a renewal-reward process which describes the outflow of capital caused by payments for claims occured in the interval $[0, t]$, $N(t)$ is a renewal process counting the total number of claims in $[0, t]$ and X_i's are i.i.d random variables denoting the amount of payment for i^{th} claim. As seen in (1), the term $\boldsymbol{ct}$ expressing the company's premium income is a linear function of time. However, this assumption is not realistic, because the premium income of an insurance company can not always increase linearly. Therefore, it is advisable to assume that the premium income is modeled as a function whose rate of growth decreases with time, although this function is monotonically increasing. For this reason, in this work, a more realistic special non-linear mathematical model is constructed and investigated, which is given as follows:

$$\boldsymbol{V(t) = u + c\sum_{i=1}^{N(t)} \ln(1 + W_i) + c\ln\big(1 + (t - T_{N(t)})\big) - S(t)} \qquad (2)$$

In (2), W_i's $(i = 1, 2, 3 \dots)$ are positive i.i.d sequence of random variables describing inter-arrival times of claims; $T_{N(t)} = \sum_{i=1}^{N(t)} W_i$ is a renewal-reward process, corresponding to the sequence of random variables $W_i's$, $i = 1,2,3, \dots$, and $V(t)$ defines company's capital balance at any time t which is modelled by a Logarithmic Risk Process. The main purpose of this study is to evaluate ruin probability of non-linear risk model (2). For this aim, Lundberg type upper bound is obtained for the ruin probability of Logarithmic Risk Model (2).

Keywords: Risk Theory, Ruin Probability, Cramér-Lundberg model, Lundberg Inequality, Non-linear Insurance Model

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On the Solvability of the Nonhomogeneous Riemann Problem in the Weighted Smirnov Classes with the General Weight

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Abstract

In this work the nonhomogeneous Riemann problem of the theory of analytic functions with a piecewise continuous coefficient in weighted Smirnov classes with a general weight is considered. The sufficient conditions on the coefficient of the problem and on the weight function are found, which disappears at an infinitely remote point.

Keywords: Riemann problem, Smirnov classes, weight function

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Samudu Transform method for evaluation of two dimensional modified fractional partial differential equation, an application to financial modeling with Islamic perspective

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Abstract

The word '*Urboūn'* is not a new Islamic term, though it has now been occasionally revealing by Muslim scholars in their studies of financial transactions. Currently a development in relation to the terms 'Option and Black-Sholes PDE equations' has become a greater part of the conventional markets. Option itself incurs the values of assets by putting and calling incidences in future planning dilemmas by using Black-Sholes PDE equations to uphold stability of stock markets in all around the world, casting those values on risk-free marketing jumbles.

This study is a serious trial of synchronizing the Arabic term '*Urboūn'*, within the future prospects of financial markets that will be reinforced by using Black-Sholes equations but will be differing by using the values of Risk-full Marketing (comprising full risk management) that is an indispensible objective of Islamic jurisprudence. Conclusively the Black-Sholes equations have been modified in this study. The solutions are obtained by using Samudu Transform for both Islamic and conventional financial market.
Keywords:'*Urboūn'*, Options, Black-Sholes, Risk management,Islamic Jurisprudence

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Josiane Correia de Souza Carvalho and others…'AN ANALYSIS OF THE DAILY VARIATION OF THE VALUE OF AN OPTION OF A SHARE THROUGH THE BLACK-SCHOLES EQUATION' manuscript published in International Journal of Applied Mathematics Volume 30 No. 3 2017, 239-251; ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version) doi: http://dx.doi.org/10.12732/ijam.v30i3.3 p/240

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Planarity of a New Class of Dembowski-Ostrom Polynomials

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Abstract

The planar mappings, which introduced as a tool to construct finite projective planes by Dembowski and Ostrom in 1968 [1], correspond to perfect nonlinear functions in cryptography [2]. In [1], Dembowski-Ostrom polynomials first were identified as important objects in the study of specific projective planes. Nowadays, these polynomials are formed by the basis of multivariate public key cryptosystems defined over finite fields, which have importance in post quantum cryptography. In the literature, planar Dembowski-Ostrom polynomials have been studied in form of the bilinear polynomials (polynomials that can be written as a product of two linearized polynomials) or nonbilinear polynomials. In [3], Özbudak and Kyureghyan investigated the planarity of bilinear Dembowski-Ostrom polynomials of the form $L_1(x).L_2(x)$. In [4], the planarity of bilinear Dembowski-Ostrom polynomials, defined in the type $f_{A,B}(x) = x(x^{q^2} + Ax^q + Bx)$ over F_{q^3}, where $A, B \in F_q$, were proposed.

In this paper, we suggest the planarity of a novel class of Dembowski-Ostrom polynomials. In order to do this, we first introduce a new class of nonbilinear Dembowski-Ostrom polynomials of the form $f(x) = x(x^{2q^2} + x^{2q} + x^2 + Ax^{q+1} + Bx^{q^2+1} + Cx^{q^2+q})$ defined over F_{q^3}, where $A, B, C \in F_q$. Based on the fact that all difference mappings must be bijective for a polynomial to define planar mappings, and the fact that the difference mappings of Dembowski-Ostrom polynomials are linearized polynomials, we use the Dickson matrices that allow us to learn whether linear polynomials are permutation polynomials. Using the relationship between the determinant polynomial obtained from Dickson matrix and the algebraic curves defined over finite fields, we will state under which conditions whether the new polynomial class is planar.

Keywords: Linearized Polynomials, Dembowski-Ostrom polynomials, Planarity

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POSTER SESSION

On the solutions of a system of (2p+1) difference equations of higher order

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Abstract

In this paper we represents the well-defined solutions of the system of the higher-order rational difference equations in terms of Fibonacci and Lucase sequences, where the initial values do not equal -3.Some theoretical explanations related to the representation for the general solution are also given.

Keywords: Closed-form formula, Lucas numbers, Fibonacci numbers, system of difference equations.

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Integral Sliding Mode Control of a DC-DC Boost Converter

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Abstract

Improving the performance of DC-DC converters is of primary interest for industrial, military and cybernetic applications. Designing controllers for systems with DC-DC converters presents interesting challenges because these systems are non-linear and time-varying. These controllers must be robust against the uncertainties, disturbances introduced and the variability of the system parameters. To meet these challenges, two types of controls were used: integral sliding mode and double integral sliding mode controls.

Sliding mode control has been applied to many systems including DC-DC boost converters, with satisfactory results. Sliding mode controllers are well known for their robustness and stability. However, employed techniques offer only asymptotic convergence and steady state error. This last weakness of sliding mode control is addressed first by integral (ISMC) and then by double integral (DISMC) sliding mode approaches in order to improve performance in precision.

Different simulations with reference and load changes were also performed to test the functionalities of such controllers. The results obtained show excellent dynamic performance of the control by integral sliding mode for a fairly wide operating range which has highlighted the non-linear nature of the controller.

Neutrosophic Triplet Rings and its Applications to Mathematical Modelling

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Abstract

Rings and fields have main importance when we study algebraic structures, as both of them are based on the group structure. In our study, we extend the concept of a neutrosophic triplet group to a neutrosophic triplet ring and a neutrosophic triplet field. We discuss a neutrosophic triplet ring and some of its properities. Moreover, we define the neutrosophic triplet subring, neutrosophic triplet ideal, and nilpotent integral neutrosophic triplet domain. Finally, we introduce a neutrosophic triplet field.

Keywords: Neutrosophic triplet structures, algebraic structure, neutrosophic triplet rings, neutrosophic triplet group.

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On a blow up property of solutions some nonlinear problem

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Abstract

In this abstract, we study the blow up for the nonlinear problem

$$u_{tt}-\sum_{i=1}^{n}D_i(|D_iu|^{p-2}D_iu)-\alpha\Delta u_t+f(u)=0,\ (x,t)\in\Omega\times(0,T), \quad (1)$$

$$u(x,0)=u_0(x),\ u_t(x,0)=u_1(x),\ \ x\in\Omega, \quad (2)$$

$$\sum_{i=1}^{n}(|D_iu|^{p-2}D_iu)\cos(x_i,\nu)+\alpha\frac{\partial u_t}{\partial n}=g(u),\ (x,t)\in\partial\Omega\times[0,T], \quad (3)$$

where $n\geq 2$, $\Omega\subset R^n$ is a domain with smooth boundary $\partial\Omega$, $u_0(x)\in W_2^1(\Omega)$, $u_1(x)\in L_2(\Omega)$ are given functions, $f(u)$ and $g(u)$ are some nonlinear functions, α is a positive number, $p\geq 2$, $D_i=\frac{\partial}{\partial x_i}$, $i=1,2,...,n$, $\frac{\partial}{\partial n}$ -denotes external normal in $\partial\Omega$.

There are lot of studies for the problem (1)-(3) (see, e.g. [1]- [5]), where mainly a nonlinearity in the equation are presented. In this work, we study a blow up problem (1)-(3), when the boundary functions are non smooth. The following assertion is pursued in this abstract.

Theorem. Let $F(u)=\int_0^u f(s)ds$, $G(u)=\int_0^u g(s)ds$ and u_0,u_1 are such that $(u_0,u_1)>0$, moreover, the next conditions are fulfilled:

$$2(2\alpha+1)F(s)-sf(s)\geq 0,\ \ sg(s)-2(2\alpha+1)G(s)\geq 0 \quad \text{as} \quad s\in(0,\infty);$$

$$\int_\Omega F(u_0)dx-\int_{\partial\Omega}G(u_0)ds+\frac{1}{p}\int_\Omega\sum_{i=1}^{n}|D_iu_o|^p dx\leq 0.$$

Let $u(x,t)\in W_2^1(0,T;W_2^2(\Omega))\cap W_2^2(0,T;L_2(\Omega))$ be a solution of problem (1)-(3). Then there exists a $t_o<\infty$ such that,

$$\lim_{t\to t_0}\left[\|u(x,t)\|^2+\int_0^t\|\nabla u(x,\tau)\|^2d\tau\right]=\infty.$$

Keywords: blow up, nonlinear equation, boundary value problem, a prior estimate. Hyperbolic equation.

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Finite time blow-up for quasilinear wave equations with nonlinear strong damping

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Abstract

In this paper we consider a class of quasilinear wave equations

$$u_{tt} - \Delta_\alpha u - \omega_1 \Delta u - \omega_2 \Delta_\beta u_t + \mu |u_t|^{m-2} u_t = |u|^{p-2} u,$$

associated with initial and Dirichlet boundary conditions. Under certain conditions on α, β, m and p we show that any solution with positive initial energy, blows up in finite time. Furthermore, a lower bound for the blow-up time will be given.

.

Keywords: Nonlinear wave equation, strong damping, blow-up**.**

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Spectral Analysis and Semigroup Generation of a Flexible Bernoulli Beam

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Abstract

A flexible Bernoulli beam with structural damping is investigated in this paper. The beam System is described by partial differential equations with initial and boundary conditions. First, the system is transferred to an abstract evolution equation in an appropriate Hilbert space, and then spectral properties and semigroup generation of system operator are studied andpresented. Finally, the exponential stability of the system is discussed and explored.

Keywords: Flexible Beam, Partial Differential Equations, Semigroup of Linear Operators, Exponential Stability

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An accurate high frequency full wave mathematical model for nanometric Silicon PIN diode

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Abstract

In recent, the effect of electromagnetic radiation on semiconductor active devices and the coupling effect which occurs at high frequency between different circuit elements become more and more important.

This paper presents a high frequency full wave model for Silicon PIN diode. Ended, we present a three dimensional solutions for the electromagnetic field equations (Maxwell's equations) considering finite difference time domain (FTDT) method to describe the circuit passive part [1,2]. So, we include the electromagnetic effect by solving Maxwell's equations while taking into account the interaction between electromagnetic wave and active device.

The electromagnetic field equations considered perfectly electronic conducting (PEC) surfaces boundary conditions to model the stripline effect. The perfectly matched layer (PML) absorbing boundary condition are also considered to truncate the FDTD lattices and decay rapidly the incident wave [3]. We consider a Gaussian pulse excitation and the Fourier Transform algorithm (FFT) in output and input time responses to obtain frequency domain comportment.

So, We propose mathematical method to couple a three-dimensional (3-D) time domain solution of Maxwell's equations to the PIN diode. This later is modeled by the active device model (DDM: Drift Diffusion Model) which provides the time and space distribution of the electrostatic potential, carriers concentration, current density for the PIN diode. On an other hand, the full wave model solves Maxwell's equations. The coupling between the two models is established by considering the electric and magnetic fields obtained from the solution of Maxwell's active device equations. To update these fields, the current densities are then computed. Numerical results are generated to investigate the effects of active device-wave interaction on the behavior of a nanometric diode PIN [4].

The active devices in the microwave circuits is typically very small in size compared to a wavelength, then it can be modeled by its equivalent lumped device with a very high degree of accuracy. Thus, in the conventional lumped element-FDTD (LE-FDTD) approach, two contacts for the active device (anode, cathode) are considered as current sources that interact with the Maxwell's equations, exactly with Maxwell– Ampere's equation, the s-parameters are extracted using the Fast Fourier transform (FFT)of time-domain results.[5,6,7,8].

Key words: FTDT, Silicon PIN, Electromagnetic, boundary conditions, current densities.

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Analysis of The Transient Stability of An Electrical Network in The Presence of SMES.

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Abstract

In this work, we have studied and analyzed the transient stability in particular for a simple electrical grid. Our study aims to highlight the importance of using the storage system SMES, to improve consumption and improve transit stability as well as to avoid power outages.

The impact of the SMES on the system stability is better than the action of the classical regulation.The optimal position of the SMES is generally close to the machines at the risk.The stabilization of the system by using one, tow and three SMES is examined.

Keywords: Transient stability, FACTS, SMES, Power converter.

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MATHEMATICAL METHODS SAFETY BARRIER PERFORMANCE ASSESSMENT

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ABSTRACT

Most well-designed systems have safety barriers against such circumstances to protect humans, the environment, and material assets. This makes it harder for any one initiating event to propagate through all the barriers culminating in a hazardous event or accident. Some barriers are set up to prevent accidents from occurring (prevention barriers). Others are in place to reduce the consequences of an event once it has already occurred (mitigation barriers). The purpose of this paper is evaluating the performance of the existing safety barriers and according to risk tolerable decides if more additional barriers should be implemented.

Key Words: *Barrier, Failure, Lopa, Tolerable risk, Safety instrumented system*

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Classification of land cover by spectral and textural characteristics

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Abstract

The rapid development of computer science and applications of the broad spectrum of programmable systems (eg, Matlab, Matmatics, Mapple, etc.) have enabled the use of satellite data in different identity problems and recognition. The application of new approaches and a comparative analysis of the results obtained with known results is of great importance, both from a theoretical and applied point of view. This article is devoted to the classification of soil plants according to spectral characteristics obtained by remote sensing methods. The problem of preliminary analysis of the informativeness of the spectral features themselves and the textural features calculated on their basis is considered. The LBP (Local binary pattern) histogram method is used to calculate the values of textural attributes. The LBP method describes the spatial structure of the image according to the local structure of the texture. In this article, the cotton field of the Hajigabul region of the Azerbaijan Republic was taken as the study area. The classification of land cover was carried out using metric distances and classification methods. The data obtained on the spectral channels (blue, green, red, and near infrared) were used as spectral data using an unmanned aerial vehicle XA-Rotor-1000 X-8. The "pdist", "linkage" and "cluster" functions included in the MATLAB batch programs were used to perform the calculations that led to the definition of which class each object belongs toIn 11 the calculations, the Euclidean distance was used as the metric distance. The "nearest neighbor" classification algorithm was applied. Given comparative mathematical approaches to the solution of the classification of plants-soil using remote data. Investigated the possibility of applying the automatic classification of soil plants according to the remote data of the classification and recognition methods included in the Matlab software system. It is shown that in the problems of classification of objects, "The statement that the objective decision rule, when a fragment is taken as a whole, more efficient than the pixel decision rule," is not always true.

Keywords: remote data, classification, metric distance, soil type, identification, recognition, textural signs.

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Hasse Prensiple and Brauer Manın Obstraction

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Abstract

Hasse principle is one of the important theorems of number theory. It enables us to reach a conclusion about the existence of rational solutions of quadratic Diophantine equations. However, this method does not work for higher degrees. In some cases the failure of Hasse principle can be explained by Brauer-Manin Obstruction. This poster explains the general steps of this process.

Keywords: Hasse Prenciple, Brauer Manin Obstraction,

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Dynamics of Rogue waves and Generalized breathers of Benjamin-Ono equation

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Joint work with K. Sakkaravarthi, K. Murugesan, R. Sakthive

Abstract

Rogue waves and breathers are one of the very interesting localized nonlinear structures. Although, the appearance of rogue waves are not limited to ocean but in finance, plasma, superfluid, Bose-Einstein condensate and well known optical rogue waves. The mathematical justification of these nonlinear structures are always an interesting task.

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On the strong solvability of the nonlinear parabolic equations

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Abstract

In this abstract, we annownce a strong solvability result for a class of nonlinear parabolic equations with principial part is heat operator and the rifht part is a given function which depend on the set of time and spatial variales, also depend on the solution and its spatial gradient. For such equation it has been proved a strong solvability result in the proper parabolic Sobolev space for the initial-boundary value problem. It is found a sufficient condition for the growth order of the right hand side in dependence of the gradient of the solution. The growth order contains a function coefficient, which also is a summable function. Using the Pohojaev's approaches from [1] it is established the proper result for the parabolic equations. The growth condition for the right hand side is a generalization of well-known Bernshtain's condition for elliptic equations. The domain is a sylindrical domain with its base is a smooth n-dimensional domain.

Keywords: parabolic equation, strong solution, growth condition, a priory estimate, heat operator, parabolic Sobolev space, smooth domain.

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An interpolation inequality for weight cases

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Abstract

In this abstract, we propose an interpolation inequality with three weights, which estimates a weighted Lebesgue norm of the function through the multiplicate of other weighted Lebesgue norms of function and its derivative of the next type

$$\|f\|_{q,v} \le C_0 A^{1/q} \|f\|_{m,\omega_1}^{1/q'} \|Df\|_{p,\omega}^{1/q}, \tag{1}$$

where $Df = \left(\partial_{x_1} f, \partial_{x_2} f, \ldots, \partial_{x_n} f\right)$ The following main result is asserted by this abstract

Theorem. *Let* $m > 0, p \ge 1, q \ge \max\left(p, m(p-1)/p\right)$ *and* $D \subseteq \Re^n$ *be a domain (may be unbounded). Let the positive measurable functions* v, ω_1 *are of* A_∞ *-Muckenhoupt's class and* $\sigma = \omega^{1-p'} \in L^{1,loc}$. *Then for the inequality (1) to hold for any function* $f \in Lip_0(\Omega)$ *it suffices that the Frostman's type condition*

$$|Q|^{1/n-1} v(Q) \sigma(Q)^{1/p'} \le A \omega_1(Q)^{(q-1)/m}$$

all over the balls $Q \in \Im$, $\Im = \{Q = Q(x,r) : x \in \Omega, 0 < r < d_\Omega\}$ *to be fulfilled, where* C_0 is a positive constant depending only on n, p, q and A_∞ -constants of the functions v, ω_1.

For the subject we refer e.g. the book [1, 2]. Such inequalities find a usefull application in study of the regularity properties of the degenerate parabolic equations.

Keywords: regularity of solutions, interpolation inequality, embedding results, Harnack's inequality, Harnack's inequality, fundamental solutions, weak solutions, parabolic equations

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On Holder regularity of the degenerated parabolic equations

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Abstract

In this abstract it has been studied the Holder continuity of the weak solutions of the degenerate parabolic equations

$$\frac{\partial}{\partial x_j}\left(a_{ij}(t,x)\frac{\partial u}{\partial x_i}\right)-\frac{\partial u}{\partial t}=0 \qquad (1)$$

with uniform degeneratacy condition

$$\frac{1}{C}(|x|^{\alpha}+t)|\xi|^2 \le a_{ij}(t,x)\xi_i\xi_j \le C(|x|^{\alpha}+t)|\xi|^2 \quad (2)$$

for $C>0$, $\forall\xi\in\Re^n,(t,x)\in D$, and D be a bounded domain in half-space $\{t<t_0\}$. It has been found a sufficient condition on the range of $\alpha\in\Re$ for the weak solutions of (1) to be Holder continues in the origin. For the subject we refer e.g. the works [1, 2]

Keywords: regularity of solutions, Holder regularity, Harnack's inequality, fundamental solutions, weak solutions, qualitative properties of solutions, a prior estimates.

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On a*-*I*-open Sets and a Decomposition of Continuity

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Abstract: In this paper, we introduce a new set namely a*-*I*-open set in ideal topological spaces. Besides, we give some properties and characterizations of it. We obtain that it is stronger than pre*-I-open set with b-open set and weaker than $\delta\beta_I$-open set. Finally, we give a decomposition of continuity by using a*-*I*-open set as stated the following:" $f:(X,\tau,I)\longrightarrow(Y,\varphi)$ is continuous if and only if it is a*-I-continuous and strongly A$_I$-continuous."

Keywords: a*-*I*-open set, Decomposition of continuity, Ideal.

1 Introduction and preliminaries

Topic of ideals in topological spaces has been studied since beginning of 20th century. It has won reputain and importance in citevai.Throughout this paper, we will denote topological spaces by (X,τ) and (Y,φ). For a subset A of a space (X,τ), the closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. It is well known that a subset A of a space (X,τ) is said to be regular open citevel if $A=Int(Cl(A))$. A subset A of a space (X,τ) is said to be δ-*open* citevel if for each $x\in A$ there exists a regular open set U such that $x\in U\subseteq A$. A is δ-*closed* citevel if $(X\text{-}A)$ is δ-*open*. The set $\{x\in X\mid x\in U\subseteq A$ for some regular open set U of $X\}$ is called the δ-interior of A and is denoted by $Int_\delta(A)$ citevel. A point $x\in X$ is called a δ-cluster point of A if $A\cap Int(Cl(V))\neq\varnothing$ for each open set V containing x. The set of all δ-cluster points of A is called the δ-closure of A and is denoted by $\delta Cl(A)$ citevel. Of course, δ-*open* sets form a topology τ^δ and then $\tau^\delta\subset\tau$ holds citevel.

An ideal I on X is defined as a nonempty collection of subsets of X satisfying the following two conditions:
(1) If $A\in I$ and $B\subset A$, then $B\in I$;
(2) If $A\in I$ and $B\in I$, then $A\cup B\in I$.

Let (X,τ) be a topological space and I an ideal on X . An ideal topological space is a topological space (X,τ) with an ideal I on X and is denoted by (X,τ,I). For a subset $A\subset X$, $A^*(I,\tau)=\{x\in X\mid U\cap A\notin I$ for each neighbourhood U of $x\}$ is called the local function of A with respect to I and τ (citekur). Throught this paper, we use A^* instead of $A^*(I,\tau)$. Besides, in citejan, authors introduced a new Kuratowski closure operator $Cl^*(.)$ defined by $Cl^*(A)=A\cup A^*(I,\tau)$ and obtained a new topology on X which is called an $*$-$topo\log y$. This topology is denoted by $\tau^*\ (I)$ which is finer than τ.

A point x in an ideal topological space is called δ_I-$cluster\ point$ of A if $Int(Cl^*(U)\cap A\neq\varnothing$ for each neighborhood U of x. The set of all δ_I-$cluster\ points$ of A is called the δ_I-$closure\ of\ A$ and will be denoted by $\delta Cl_I(A)$ citey/"uk. A is said to be δ_I-$closed$ citey/"uk if $A=\delta Cl_I(A)$. Of course, the complement of δ_I-open set is said δ_I-$closed$ citey/"uk. The family of all δ_I-$open$ sets in any ideal topological space (X,τ,I) form a topology $\tau^{\delta I}$ and then $\tau^{\delta I}\subset\tau$ holds citey/"uk.

Definition 1. *some label A subset A of an ideal topological space (X,τ,I) is said to be α-open citenja (resp. semi-open citelev, pre-open citemas1, b-open citeand (or γ open citeel-a), β-open citeabd) if $A\subset Int(Cl(Int(A)))$ (resp. $A\subset Cl(Int(A))$, $A\subset Int(Cl(A))$, $A\subset Int(Cl(A))\cup Cl(Int(A))$, $A\subset Cl(Int(Cl(A)))$).*

Definition 2. *some label A subset A of an ideal topological space (X,τ,I) is said to be pre-I-$open$ citedon (resp. $semi$-I-$open$ citehat1, α-I-$open$ citehat1, b-I-$open$ citeg/"ul, β-I-$open$ citehat 1) if $A\subset Int(Cl^*(A))$ (resp. $A\subset Cl^*(Int(A))$, $A\subset Int(Cl^*(Int(A)))$, $A\subset Cl^*(Int(A))\cup Int(Cl^*(A))$, $A\subset Cl(Int(Cl^*(A)))$).*

Definition 3. *A subset A of an ideal topological space (X,τ,I) is said to be δ-α-I-$open$ citehat 4, pre*-I-open citeeki (resp. semi*-I-open , $\delta\beta$-I-open citehat 4) if $A\subset Int(Cl(\delta Int_I(A)))$ (resp. $A\subset Int(\delta Cl_I(A))$, $A\subset Cl(\delta Int_I(A))$, $A\subset Cl(Int(\delta Cl_I(A)))$).*

Related to above definitions, one can find the following diagram in citehat 4. None of these implications are reversible in generally as shown in the related papers.

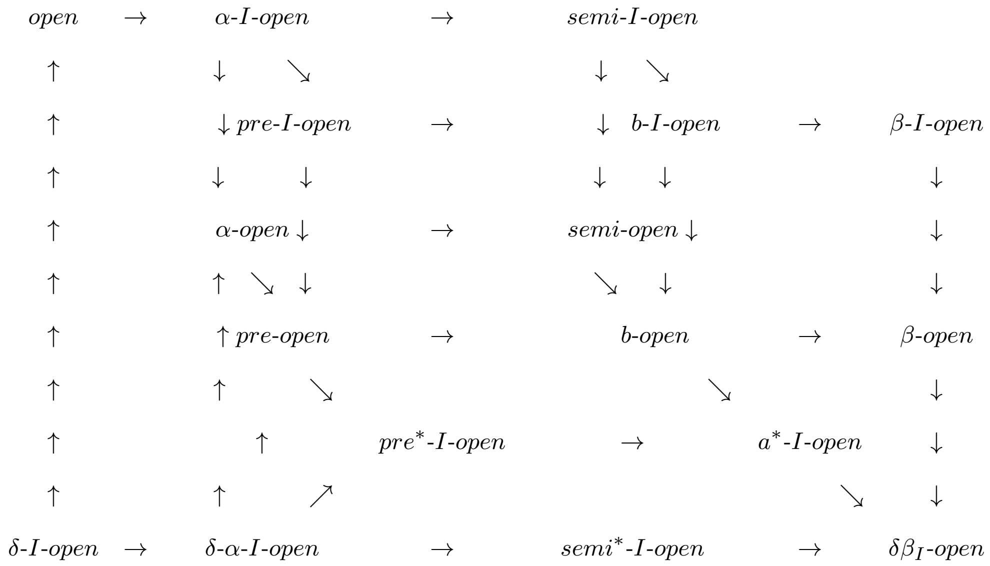

Diagram II

Lemma 1. *For a subset A of an ideal topological space (X, τ, I) , the following properties are hold:*
(1) If U is an open set, then $U \cap Cl^*(A) \subseteq Cl^*(U \cap A)$ citehat2,
(2) If U is an open set, then $\delta Cl_I(U)) = Cl(U)$ citehat3.

2 a*-*I*-open sets

In this section, to give a decomposition of open set we introduce a new set which name is a*-*I*-open set and obtain some properties and characterizations of it.

Definition 4. *A subset A of an ideal topological space (X, τ, I) is said to be an a*-I-open if $A \subset Int(\delta Cl_I(A)) \cup Cl(Int(A))$. The complement of an a*-I-open set said to be an a*-I-closed. It is obvious that A is an a*-I-closed if and only if $Cl(\delta Int_I(A)) \cap Int(Cl(A)) \subset A$.*

Corollary 1. *It is obtained from Definition 4, $\varnothing$ and X are both a*-I-open sets and a*-I-closed sets.*

Proposition 1. *Let (X, τ, I) be an ideal topological space. Then, the following properties are hold:*
(1) If A is pre-I-open, then it is a*-I-open,*
(2) If A is b-open, then it is a-I-open,*
(3) If A is a-I-open, then it is $\delta\beta_I$-open.*

Proof: The proof of (1) is clear from Definitions 1, 3 and 4. The others are obtained by using related set definitions.The following diagram is obtained by using Proposition 3 and several sets defined above. □

Remark 1. *The converses of each statements in Proposition 3 are not true in generally as shown in the next examples.*

Example 1. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\varnothing\}$.(1) Set $A = \{a, d\}$. Then, A is an a*-I-open but it is not pre*-I-open (2) Set $A = \{a, b\}$. Then, A is an a*-I-open but it is not b-open.*

Example 2. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{c\}, \{a, c\}\}$ and $I = \{\varnothing\}$. For $A = \{b, d\}$ is $\delta\beta_I$-open, but it isn't a*-I-open.*

We have the following diagram.

open	$\rightarrow$	α-*I-open*	$\rightarrow$	*semi-I-open*		
$\uparrow$		$\downarrow \quad \searrow$		$\downarrow \quad \searrow$		
$\uparrow$		$\downarrow$ *pre-I-open*	$\rightarrow$	$\downarrow$ *b-I-open*	$\rightarrow$	β-*I-open*
$\uparrow$		$\downarrow \quad \downarrow$		$\downarrow \quad \downarrow$		$\downarrow$
$\uparrow$		α-*open* $\downarrow$	$\rightarrow$	*semi-open* $\downarrow$		$\downarrow$
$\uparrow$		$\uparrow \quad \searrow \quad \downarrow$		$\searrow \quad \downarrow$		$\downarrow$
$\uparrow$		$\uparrow$ *pre-open*	$\rightarrow$	*b-open*	$\rightarrow$	β-*open*
$\uparrow$		$\uparrow \quad \searrow$				$\swarrow$
$\uparrow$		$\uparrow$	*pre**-*I-open*	$\rightarrow$	$\delta\beta_I$-*open*	
$\uparrow$		$\uparrow \quad \nearrow$		$\nearrow$		
δ-*I-open*	$\rightarrow$	δ-α-*I-open*	$\rightarrow$	*semi**-*I-open*		

Diagram I

Proposition 2. *For an ideal topological space (X, τ, I) and a subset A of X, the following property is hold: "If $I = \wp(X)$, then A is an a^*-I-open if and only if A is an b-open."*

Proof: Since sufficiency is stated in Proposition 3(2), we prove only necessity. Let $I = \wp(X)$. Then, $A^* = \varnothing$ and $Cl^*(A) = A \cup A^* = A$ for every subset A of X. So, we have $\delta Cl_I(A) = Cl(A)$. If A is an a^*-I-*open* set, then we obtain that $A \subset Int(\delta Cl_I(A)) \cup Cl(Int(A)) \subset Int(Cl(A)) \cup Cl(Int(A))$ and hence every a^*-I-*open* set is a *b-open*. □

Remark 2. *The notions of a^*-I-open set and β-open set are independent each other. Indeed in Example 2, set $A = \{b, d\}$ is β-open, but it isn't a^*-I-open. Besides in Example 1(2), set $A = \{a, b\}$ is an a^*-I-open but it is not β-open.*

Proposition 3. *Let (X, τ, I) be an ideal topological space with an arbitrary index set Δ. If $\{A_\alpha : \alpha \in \Delta\} \subset a^*IO(X, \tau)$, then $\cup\{A_\alpha : \alpha \in \Delta\} \in a^*IO(X, \tau)$.*

Proof: Since $\{A_\alpha : \alpha \in \Delta\} \subset a^*IO(X, \tau)$, $A_\alpha \subset Int(\delta Cl_I(A_\alpha)) \cup Cl(Int(A_\alpha))$ for every $\alpha \in \Delta$. Since δCl_I is a Kuratowski closure operator, we have

$$\begin{aligned}
(\bigcup_{\alpha \in \triangle} A_\alpha) &\subset \left(\bigcup_{\alpha \in \triangle} Int(\delta Cl_I(A_\alpha)) \cup Cl(Int(A_\alpha)) \right) \\
&= \left(\bigcup_{\alpha \in \triangle} Int(\delta Cl_I(A_\alpha)) \right) \cup \left(\bigcup_{\alpha \in \triangle} Cl(Int(A_\alpha)) \right) \\
&\subset Int \left(\bigcup_{\alpha \in \triangle} \delta Cl_I(A_\alpha) \right) \cup Cl \left(\bigcup_{\alpha \in \triangle} Int(A_\alpha) \right) \\
&\subset Int(\delta Cl_I \left(\bigcup_{\alpha \in \triangle} A_\alpha \right) \cup Cl(Int \left(\bigcup_{\alpha \in \triangle} A_\alpha \right).
\end{aligned}$$

□

Proposition 4. *Let (X, τ, I) be an ideal topological space and A, U are subsets of X. If A is an a^*-I-open set and U is δ-I-open set. Then $(A \cap U)$ is an a^*-I-open set.*

Proof: Since A is an a^*-I-*open* set and U is δ-I-*open* set, we have $A \subset Int(\delta Cl_I(A)) \cup Cl(Int(A))$ and $U \subset \delta Int(U)$. By using some properties of closure, interior and δ-I-closure operations, we have

$$\begin{aligned}
(A \cap U) &\subset ((Int(\delta Cl_I(A))) \cup Cl(Int(A))) \cap \delta Int_I(U) \\
&= (Int(\delta Cl_I(A)) \cap \delta Int_I(U)) \cup (Cl(Int(A)) \cap \delta Int_I(U)) \\
&\subseteq (Int(\delta Cl_I(A)) \cap Int(U)) \cup (Cl(Int(A)) \cap Int(U)) \\
&\subseteq Int\,[\delta Cl_I(A) \cap Int(U)] \cup Cl\,[Int(A) \cap Int(U)] \\
&\subseteq Int\,(\delta Cl_I\,(A \cap Int(U))) \cup Cl(Int(A \cap U)) \\
&\subseteq Int\,(\delta Cl_I\,(A \cap U)) \cup Cl(Int(A \cap U)).
\end{aligned}$$

This shows that $(A \cap U)$ is an a^*-I-*open* set. □

Definition 5. *A subset A of an ideal topological space (X, τ, I) is called*
(1) strongly t-I-set citeeki if $Int(\delta Cl_I\,(A) = Int\,(A)$,
(2) strongly A_I-set if $A = U \cap V$, where $U \in \tau$ and V is strongly t-I-set and $Int(\delta Cl_I\,(V) = Cl\,(Int\,(V))$.

Theorem 1. *The following properties hold for a subset A of an ideal topological space (X, τ, I):*
(1) If A is strongly t-I-set and $Int(\delta Cl_I\,(A) = Cl\,(Int\,(A))$, then it is strongly A_I-set,
(2) If A is open set, then it is strongly A_I-set.

Proof:
(1) : Since A is strongly t-I-set with $Int(\delta Cl_I\,(A) = Cl\,(Int\,(A))$ and $X \in \tau$, the proof of 1) is obvious.
(2) : Since X is strongly t-I-set with $Int(\delta Cl_I\,(X) = Cl\,(Int\,(X))$ and $A \in \tau$, the proof of 2) is obtained. □

Theorem 2. *For a subset A of (X, τ, I), the following properties are equivalent:*
(1) A is open,
(2) A is an a^*-I-open and strongly A_I-set.

Proof: $(1) \Longrightarrow (2)$: By Diagram II, every open set is a^*-I-$open$. Besides, we have every open set is strongly A_I-set according to Theorem 7(2).

$(2) \Longrightarrow (1)$: Let A is an a^*-I-$open$ and strongly A_I-set. Then, we have $A \subset Int(\delta Cl_I(A)) \cup Cl(Int(A))$ and strongly A_I-set if $A = U \cap V$, where $U \in \tau$ and V is strongly t-I-set and $Int(\delta Cl_I\,(V) = Cl\,(Int\,(V))$, respectively. Therefore, we have $A \subset Int(\delta Cl_I(U \cap V)) \cup Cl(Int(U \cap V)) \subseteq [Int(\delta Cl_I(U)) \cap Int(\delta Cl_I(V))] \cup [Cl(Int(U) \cap Cl(Int(V)] = [Int(\delta Cl_I(U)) \cap Cl(Int(V)] \cup [Cl(Int(U) \cap Cl(Int(V)]$. According to Lemma 1(2), since $U \in \tau$, it is obvious that $\delta Cl_I(U) = Cl(U)$ and $Int(\delta Cl_I(U)) = Int(Cl(U))$. So, we have
$A \subset [Int(Cl(U)) \cap Cl(Int(V)] \cup [Cl(Int(U) \cap Cl(Int(V)] = [Int(Cl(U)) \cup Cl(Int(U)] \cap Cl(Int(V)$. Consequently, since $A \subset U$, we obtain $A \subset U \cap \{[Int(Cl(U)) \cup Cl(Int(U)] \cap Cl(Int(V))\} = \{U \cap [Int(Cl(U)) \cup Cl(Int(U)]\} \cap Cl(Int(V)) = [(U \cap Int(Cl(U))) \cup (U \cap Cl(Int(U)))] \cap Cl(Int(V)) = U \cap Int(V) = Int(U \cap V) = Int(A)$. Hence A is an open. □

The notions of a^*-I-$open$ set and strongly A_I-set are independent each other as shown in the following examples.

Example 3. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ and $I = \{\varnothing, \{d\}\}$. For $A = \{a\}$, then it is a^*-I-open but it isn't strongly A_I-set.*

Example 4. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{c\}, \{a, c\}\}$ and $I = \{\varnothing, \{a\}\}$. For $A = \{b, d\}$, then it is strongly A_I-set but it isn't a^*-I-open.*

3 Decomposition of continuity

In this section, we introduce the notions of a^*-I-continuity, strongly A_I-continuity and obtain a decomposition of continuity.

Definition 6. *A function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is said to be b-continuous citeel-a if $f^{-1}\,(V)$ is a b-open set in (X, τ) for every open set V in (Y, φ).*

Definition 7. *A function $f : (X, \tau, I) \longrightarrow (Y, \varphi)$ is said to be pre*-I-continuous citeeki (resp. δ $beta_I$-continuous citehat4, a^*-I-continuous strongly A_I-continuous) if $f^{-1}\,(V)$ is a pre*-I-open (resp. $\delta\beta_I$-open, a^*-I-open set, strongly A_I-set).*
(resp. $\delta\beta_I$-open, a^*-I-open set, strongly A_I

Proposition 5. *For a function f:$(X, \tau, I) \longrightarrow (Y, \varphi)$, the following properties are hold: (1) If f is pre*-I-continuous, then f is a^*-I(2) If f is b-continuous, then f is a^*-I-continuous, (3) If f is a^*-I-continuous, then f is $\delta\beta_I$*

Proof: The proofs are omitted from Proposition 3 as consequences by using Definitions 6 and 7. □

Remark 3. *The converses of each statements in Proposition 9 are not true in generally as shown in the next examples.*

Example 5. *Let (X, τ, I) be an ideal topological space as same as in Example 1 and $Y = \{a, b\}$, $\varphi = \{Y, \varnothing, \{a\}\}$. (1) Let $f : (X, \tau, I) \longrightarrow (Y, \varphi)$ be a function defined as $f(a) = f(d) = a$, $f(b) = f(c) = b$. Then f is a^*-I-continuous, but it isn't pre*-I-continuous.*
(2) Let $f : (X, \tau, I) \longrightarrow (Y, \varphi)$ be a function defined as $f(b) = f(d) = a$, $f(a) = f(c) = b$. Then f is a^*-I-continuous, but it isn't b-continuous.

Example 6. *Let (X, τ, I) be an ideal topological space as same as in Example 2 and $Y = \{a, b\}$, $\varphi = \{Y, \varnothing, \{a\}\}$. Let $f : (X, \tau, I) \longrightarrow (Y, \varphi)$ be a function defined as $f(a) = f(d) = a$, $f(b) = f(c) = b$. Then f is $\delta\beta_I$-continuous, but it isn't a^*-I-continuous.*

It is known that a function $f : (X, \tau) \longrightarrow (Y, \varphi)$ is continuous if $f^{-1}\,(V)$ is an open set in (X, τ) for every open set V in (Y, φ).

Theorem 3. *For a function $f : (X, \tau, I) \longrightarrow (Y, \varphi)$, the following statements are equivalent: (1) f is continuous, (2) f is a^*-I-continuous and strongly A_I-continuous.*

Proof: This follows from Theorem 8. □

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Geometric Interpretation of Curvature Circles in Minkowski Plane

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Abstract: In this study, we investigate the geometric interpretation of the curvature circles of motion at the initial position in Minkowski plane. We consider the equations of the circling-point and centering-point curves of one-parameter motion in Minkowski plane and then determine the positions of these curves relative to each other.

Keywords: Centering-point curve, Circling-point curve, Minkowski plane.

1 Introduction

The concept of instantaneous invariants was first given by Bottema to determine the geometric properties of a moving rigid body at a given moment. Therefore, the geometric and kinematic properties of planar motions in Euclidean space are investigated according to these invariants [1] and this method has also guided many studies in the field of kinematics [2–6]. Later, the instantaneous invariants were called B-invariants (Bottema-invariants) by Veldkamp [7]. Besides, Veldkamp found special geometrical ground curves such as the inflection curve, the circling-point curve and the centering-point curve with the help of B-invariants, as well as the intersection points of these curves, Ball and Burmester points [8, 9]. The special geometrical ground curves in Minkowski (Lorentz) plane and their intersection points were analyzed by recent studies [10, 11], however, the positions of these curves relative to each other have not been studied yet. Therefore, it is aimed to present the geometric interpretation of curvature circles relative to each other throughout one-parameter planar motion in Minkowski plane based on the above-mentioned studies.

2 Preliminaries

The Minkowski plane L is the plane R^2 endowed with the Lorentzian scalar product given by $\langle x, y\rangle = x_1y_1 - x_2y_2$, where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. The norm of a vector is defined by $\|x\| = \sqrt{|\langle x, x\rangle|}$. An arbitrary vector $x \in L$ is called timelike if $\langle x, x\rangle < 0$, spacelike if $\langle x, x\rangle > 0$ or $x = 0$, lightlike if $\langle x, x\rangle = 0$ whereby $x \neq 0$. Two vectors x and y are said to be orthogonal if $\langle x, y\rangle = 0$. Let L_m be a Minkowski plane in continuous motion relative to a fixed Minkowski plane L_f. Then one-parameter planar motion L_m with respect to L_f is represented by

$$\begin{aligned} X &= x\cosh\theta + y\sinh\theta + a \\ Y &= x\sinh\theta + y\cosh\theta + b \end{aligned} \tag{1}$$

with respect to Cartesian frames of reference xoy and XOY in L_m and L_f, respectively. Here $a,\ b$ and θ are functions depending on time t. The position corresponding to $\varphi = 0$ of L_m is called initial position. The values for the initial position of the nth $(n = 0, 1, 2, \ldots)$ derivative of a function f of φ with respect to φ is denoted by f_n.

The Minkowski plane L_m is chosen to rotate with a constant angular velocity relative to the fixed Minkowski plane L_f, that is, $\theta = t$. The canonical relative system of motion is constructed by

$$a_0 = b_0 = a_1 = b_1 = a_2 = 0 \tag{2}$$

and the instantaneous invariants a_n and b_n characterize completely the infinitesimal properties of motion of Minkowski planes up the n-th order as

$$\begin{aligned} &X = x, \quad X' = y, \quad X'' = x, \quad X''' = y + a_3, \\ &Y = y, \quad Y' = x, \quad Y'' = y + b_2, \quad Y''' = x + b_3 \end{aligned} \tag{3}$$

for $t = 0$ [10, 11].

3 The curvature circles in Minkowski plane

In this section, let's first recall the definitions of curvature circles in Minkowski plane.

Definition 1. *The locus of the points of moving Minkowski plane L_m, whose curvature of the trajectory is constant at initial position, is called circling-point curve in Minkowski plane and denoted by cp.*

The equation of the circling-point curve *cp* in Minkowski plane is

$$\left(x^2-y^2\right)(a_3x-b_3y)+3x\left(x^2-y^2+y\right)=0,\quad (x,y)\neq(0,0) \tag{4}$$

where $(x,y)\neq(0,0)$ or $x\neq\mp y$, [10, 11].

Definition 2. *The locus of the curvature centers of the points of moving Minkowski plane L_m is called centering-point curve in Minkowski plane and denoted by $c\tilde{p}$.*

The equation of the centering-point curve $c\tilde{p}$ in Minkowski plane is

$$\left(x^2-y^2\right)(a_3x-b_3y)+3xy=0 \tag{5}$$

where $(x,y)\neq(0,0)$ or $x\neq\mp y$, [10, 11].

Now, let us examine the positions of circling-point and centering-point curves relative to each other in Minkowski plane. The curve *cp* given by equation (4) and the curve $c\tilde{p}$ given by equation (5) can be arranged as

$$\left(x^2-y^2\right)\left(\tfrac{(a_3+3)}{3}x-\tfrac{b_3}{3}y\right)+xy=0$$

and

$$\left(x^2-y^2\right)\left(\tfrac{a_3}{3}x-\tfrac{b_3}{3}y\right)+xy=0,$$

respectively.

On the other hand, a third-order cubic curve γ in Minkowski plane can be given by

$$(\alpha x+\beta y)\left(x^2-y^2\right)+xy=0. \tag{6}$$

Let γ be an irreducible curve, this means that $\alpha\beta\neq0$.

If $\alpha=\frac{a_3+3}{3}$ and $\beta=-\frac{b_3}{3}$ are satisfied, then the curve given by the equation (6) corresponds to the circling-point curve *cp* according to the canonical system in Minkowski plane.

Moreover, if there are the relations $\alpha=\frac{a_3}{3}$ and $\beta=-\frac{b_3}{3}$, then the curve given by the equation (6) corresponds to the centering-point curve $c\tilde{p}$ according to the canonical system in Minkowski plane.

Theorem 1. *The parametric equation of the curve γ is given by*

$$x=\frac{u}{(u^2-1)(\alpha+\beta u)},\quad y=\frac{u^2}{(u^2-1)(\alpha+\beta u)} \tag{7}$$

where $u\neq\pm1$.

Proof: If we substitute $y=ux$, such that $u\neq\pm1$, in the equation (6), then we get $x^3(\alpha+\beta u)\left(1-u^2\right)+ux^2=0$. Afterwards, some direct calculations completes the proof. □

Specifically, the parametric value $\frac{-\alpha}{\beta}$ corresponds to the infinity point of the curve γ. We can examine the reducible states of this curve in the following corollaries:

Corollary 1. *In Minkowski plane, the parametric equation of the curvature circle Γ_0, which is tangent to the curve γ along the axis y, is represented by*

$$x=\frac{1}{\beta(u^2-1)},\quad y=\frac{u}{\beta(u^2-1)}. \tag{8}$$

Proof: If $\alpha=0$ is taken in the equation (7) then the proof is obvious. □

Corollary 2. *In Minkowski plane, the parametric equation of the curvature circle Γ_1, which is tangent to the curve γ along the axis x, is given by*

$$x=\frac{u}{\alpha(u^2-1)},\quad y=\frac{u^2}{\alpha(u^2-1)}. \tag{9}$$

Proof: Taking $\beta=0$ in the equation (7) completes the proof. □

From the equation (8), the Cartesian equation of the curvature circle Γ_0 in Minkowski plane is represented as

$$\beta\left(x^2-y^2\right)+x=0. \tag{10}$$

Similarly, by taking the equation (9) the Cartesian equation of the curvature circle Γ_1 in Minkowski plane is given by

$$\alpha\left(x^2-y^2\right)+y=0. \tag{11}$$

Let the points $A_i\ (i=1,2,3)$ be on the curve γ. In that case, these points are given as

$$A_i=\left(\frac{u_i}{\left(u_i{}^2-1\right)\left(\alpha+\beta u_i\right)},\quad \frac{u_i{}^2}{\left(u_i{}^2-1\right)\left(\alpha+\beta u_i\right)}\quad\right),\quad (i=1,2,3).$$

Theorem 2. *The points $A_i\ (i=1,2,3)$ with parametric value $u_i\ (i=1,2,3)$ are on the same line does not pass through the origin if and only if*

$$u_3u_2u_1=\frac{\alpha}{\beta}. \tag{12}$$

Proof: The points A_i are on the same line that does not pass through the origin if and only if the slopes of the lines A_1A_2 and A_2A_3 are equal the each other. Thus, there is the relationship

$$\frac{\frac{-u_3{}^2}{(1-u_3{}^2)(\alpha+\beta u_3)}+\frac{u_2{}^2}{(1-u_2{}^2)(\alpha+\beta u_2)}}{\frac{-u_3}{(1-u_3{}^2)(\alpha+\beta u_3)}+\frac{u_2}{(1-u_2{}^2)(\alpha+\beta u_2)}}=\frac{\frac{-u_2{}^2}{(1-u_2{}^2)(\alpha+\beta u_2)}+\frac{u_1{}^2}{(1-u_1{}^2)(\alpha+\beta u_1)}}{\frac{-u_2}{(1-u_2{}^2)(\alpha+\beta u_2)}+\frac{u_1}{(1-u_1{}^2)(\alpha+\beta u_1)}}.$$

In this manner, we get

$$\beta^2u_1u_2{}^2u_3+\beta\alpha\left(u_2\left(u_1u_3-1\right)\right)-\alpha^2.$$

If this equation is factored, we find

$$\left(\beta u_1u_2u_3-\alpha\right)=0 \text{ or } \left(\beta u_2+\alpha\right)=0.$$

So, we can write

$$u_1u_2u_3=\tfrac{\alpha}{\beta} \text{ or } u_2=\tfrac{-\alpha}{\beta}.$$

Here $u_2\neq\frac{-\alpha}{\beta}$ must be satisfied since the parametric value $\frac{-\alpha}{\beta}$ corresponds to the infinity point of the curve γ. □

If one of these three points is at the infinity, i.e., $u_3^*=\frac{-\alpha}{\beta}$, this means that this line is parallel to the asymptotes of the curve γ and cuts the curve at two points with the parameters u_1^* and u_2^*. Then the correlation between the parameters u_1^* and u_2^* is given by

$$u_1^*u_2^*=-1. \tag{13}$$

If the points A_1 and A_2 of the curve γ are represented with respect to the parameters u_1 and u_2, then the equation of the line A_1A_2 is found as

$$\left(\alpha\left(u_2+u_1\right)+\beta u_1u_2\left(u_1u_2+1\right)\right)x-\left(\alpha\left(u_1u_2+1\right)+\beta u_1u_2\left(u_2+u_1\right)\right)y+u_1u_2=0. \tag{14}$$

After the formation this equation we have

$$\alpha\left(\left(u_1+u_2\right)x-\left(u_1u_2+1\right)y\right)-\beta u_1u_2\left(-\left(u_1u_2+1\right)x+\left(u_2+u_1\right)y-\frac{1}{\beta}\right)=0. \tag{15}$$

If we denote the slopes of the lines d_1 and d_2 given by the equations

$$\left(u_1+u_2\right)x-\left(u_1u_2+1\right)y=0 \tag{16}$$

and

$$-\beta\left(u_1u_2+1\right)x+\beta\left(u_2+u_1\right)y-1=0 \tag{17}$$

by m_{d_1} and m_{d_2}, respectively, we see that these lines are perpendicular in Minkowski plane since there is the relationship $m_{d_1}m_{d_2}=1$. Hence, we can interpret that the line given by the equation (14) passes through the intersection of the lines d_1 and d_2 which are perpendicular to each other in the Minkowski plane.

Also, considering the equation of distance from a point to a line in the Minkowski plane we find the equation of the distance from origin to the line A_1A_2 as

$$d=\frac{\left|u_1u_2\right|}{\sqrt{\left|\left(-\alpha^2+\beta^2u_1^2u_2^2\right)\left(u_1^2-1\right)\left(u_2^2-1\right)\right|}} \tag{18}$$

where $u_i\neq\pm1,\ i=1,2$.

Let A_3 be a point with the parameter $-u_1$ on the curve γ. From the equation (18), the lines A_2A_1 and A_2A_3 have equal distance from origin, that is, the lines A_2A_1 and A_2A_3 are symmetrical according to the point A_2.

Now let's give the formation of the circles Γ_0 and Γ_1. Since the geometric location of the curvature centers of the curve cp is the centering-point curve $\tilde{cp}$, the curvature center of a point with the parameter u of the curve cp coincides with the same parameter point of the curve $\tilde{cp}$, [11]. Let A_1 and A_2 be two points on the curve cp. Also, let α_1 and α_2 be the centers of curvature of these points. If the points A_1 and A_2 are given by the parameters u_1 and u_2 , respectively, the equation of line A_1A_2 is found by writing $\alpha = \frac{a_3+3}{3}$, $\beta = -\frac{b_3}{3}$ in the equation (14) and the equation of line $\alpha_1\alpha_2$ is found by writing $\alpha = \frac{a_3}{3}$, $\beta = -\frac{b_3}{3}$ in the equation (14).

Thus, we get the equations of A_1A_2 and $\alpha_1\alpha_2$ lines as

$$((3+a_3)(u_1+u_2) - b_3u_1u_2(1+u_1u_2))\,x - ((3+a_3)(1+u_1u_2) - b_3u_1u_2(u_1+u_2))\,y - 3u_1u_2 = 0$$

and

$$(a_3(u_1+u_2) - b_3u_1u_2(1+u_1u_2))\,x - (a_3(1+u_1u_2) - b_3u_1u_2(u_1+u_2))\,y - 3u_1u_2 = 0,$$

respectively. Here, the lines A_1A_2 and $\alpha_1\alpha_2$ pass through the intersection of the lines given by the equations (16) and (17), which are perpendicular to each other in the Minkowski plane. Here, the equation (16) indicates a line and this line passes through the pole point P and the intersection point Q of the lines $\alpha_1\alpha_2$ and A_1A_2. The equation (17) refers to the equation of the line perpendicular to the line PQ passing through the point Q.

In case of $\alpha = 0$, by substituting the parameter equation (18) into the equation (17), for Γ_0 we get

$$u^2 - (u_2+u_1)\,u + u_1u_2 = 0. \tag{19}$$

Corollary 3. *u_1 and u_2 (the roots of the equation (19)) give the parametric expression of the intersection points of circle Γ_0 with the line given by the equation (17).*

In addition, these points are on the PA_1 and PA_2 lines. Similarly, the above statements can be investigated for the curvature circle Γ_1 in Minkowski plane. For this, let's first examine the line passing through the pole point P perpendicular to the line PQ. This line is given by the following equation taking into consideration the equation (16) such that the product of the slopes of these lines is 1 and these lines pass from pole P:

$$(u_1+u_2)\,y - (u_1u_2+1)\,x = 0.$$

If the above equation and (14) are considered together, the intersection point (is denoted by R) of this line with line A_1A_2 is on the line below

$$\alpha\,((u_1+u_2)\,x - (u_1u_2+1)\,y) + u_1u_2 = 0. \tag{20}$$

So the line passing through the point R is parallel to the line PQ. By substituting the parameter equation of circle Γ_1 into the equation (20), we get

$$u^2 - (u_2+u_1)\,u + u_1u_2 = 0. \tag{21}$$

The equation (21) is the previously obtained equation (19).

Corollary 4. *u_1 and u_2 (the roots of the equation (21)) give the parametric expression of the intersection point of the circle Γ_1 and the line given by equation (20).*

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The Measurement of Success Distribution with Gini Coefficient

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Abstract: The aim of this study is to calculate and examine the distribution of the academic success of the students of the Faculty of Education academic years between 2014-2019 the courses of statistics and probability with lorenz curve and Gini coefficient. In this regard, Tomul, E [**?**] in Educational Inequality in Turkey: Gini to Evaluate According to the index, Erdem, E., Çoban, S. [**?**] 'provinces in Turkey in Measurement Based Education Inequality and Economic Development Relationship with Difference: Education Gini Explained with coefficients.

Keywords: Academic achievement, Gini coefficient, Lorenz diagram.

1 The importance of the study

The Gini coefficient, developed by the Italian Statistician Corrado Gini (1912), is also used to determine the inequality in economic literature [**?**] because it shows simplicity and distribution with a single coefficient [**?**] and the Gini Coefficient, which is a tool used to measure inequality, in different disciplines including health and education. [**?**].

Gini coefficient of 0 means absolute equality and a value of 1 means absolute inequality. Therefore, decreasing and increasing the coefficient over time indicates the decrease and increase of inequality. In this context; What is the success inequality of Gini Coefficient and how it is distributed according to the lessons and years?

The main questions that the study seeks to answer are: What does the Gini Coefficient Achievement Distribution of Academic Achievement of Elementary Mathematics Teacher Statistics Probability course mean between the academic years of 2014-2019?

The empirical data used in the study may vary in academic terms. The sample of the study was; the academic years between 2014-2019 consists of the number of students. The number of samples between 2014-2019 is the academic achievement data of 112 students. The sample distribution by year is 2014; 36 students, 2015; 16, students, 2016; 7 students, 2017; 6 students, 2018; 28 students, 2019; 19 students

Classes	2014	2015	2016	2017	2018	2019
Statistics and Probability	36	16	7	6	28	19

The Lorenz curve examines the relationship between a certain cumulative share of national income and the cumulative share of those who obtain it. The Lorenz curve is conceptually similar to the percentage slicing method; it relates the cumulative share of income to the cumulative share of individuals, rather than simply determining their share of income. The Lorenz curve is a graphical form that shows how much the percentage income groups receive from the income distribution [**?**]. However; the usefulness of the Lorenz curve helps us to present the inequality in income distribution by a single number, without needing to tell us how much the percentage of individual groups receive.

Gini Coefficient is a non-negative number less than 1. By calculating the area between the Lorenz curve and the 45-degree line giving full equality, a numerical value ranging from 0 to 1, namely the "Gini Coefficient", is found. Where the income distribution is most fair, A = 0. The closer the Gini Coefficient is to 0, the more fair the income distribution is. Family structure of the society, population structure, educational level, tax situation, the structure of the financial sector or industry and development indicators are some factors that may affect the income distribution in a country. In general, the Gini Coefficient, i.e. the income distribution, is interpreted as sufficient after 0.40 and worse after 0.50 [**?**].

In this study, Lorenz curve and the Gini coefficient previously used in an unused area, in the area of measurement and the evaluation of the final stage of evaluation. Between the academic years of 2014 and 2019, the Faculty of Education Mathematics Education in Primary Education teacher candidates Statistics Probability courses of academic achievement was evaluated as data notes.

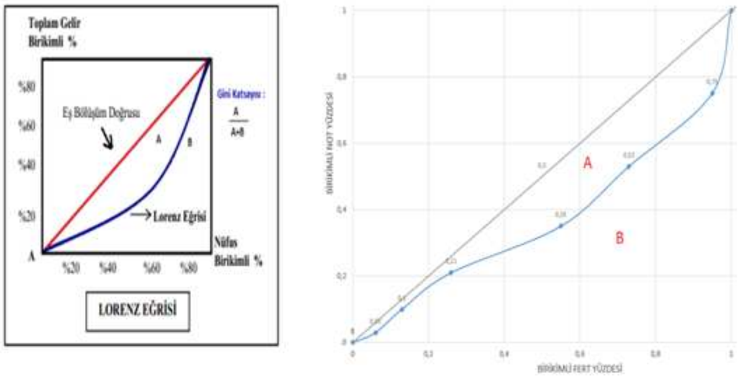

Fig. 1: Lorentz calculation chart of income and academic achievement

The Gini Coefficient Calculation of the Student Success of the year 2014 in Excel

Success Points	fi(student Frequency)	Number of Cumulative Students	percentage of cumulative students	Si(average grade)	cumulative average grade	cumulative grade average percentage	A
0	0	0	0	0	0	0	0
4≤ <17	2	2	0,06	10,5	10,5	0,03	0,000841751
17≤ <30	2	4	0,11	23,5	34	0,10	0,00356742
30≤ <43	4	8	0,22	36,5	70,5	0,203463203	0,01675485
43≤ <56	11	19	0,53	49,5	120	0,346320346	0,083994709
56≤ <69	7	26	0,72	62,5	182,5	0,526695527	0,084876543
69≤ <82	7	33	0,92	75,5	258	0,744588745	0,123597082
82≤ <95	3	36	1,00	88,5	346,5	1	0,072691198
							0,386323553

B	0,386323553
A	0,113676447
A+B	0,5

GÍNÍ=A/(A+B)=**0,22**7352894

In the results of the study, the academic achievement obtained with the Gini coefficient approach of the Elementary Mathematics Teacher Statistics Probability courses were distributed in the most fair year by year 2014 academic year, and in 2015 it moved away from the fair distribution (gini coefficient; 0.45). , 19 and 0.15 academic achievement (Gini Coefficient in general, i.e. the income distribution, up to 0.40 sufficient, 0.50 are interpreted as bad after we see).

The Geogbra Calculation of 2014

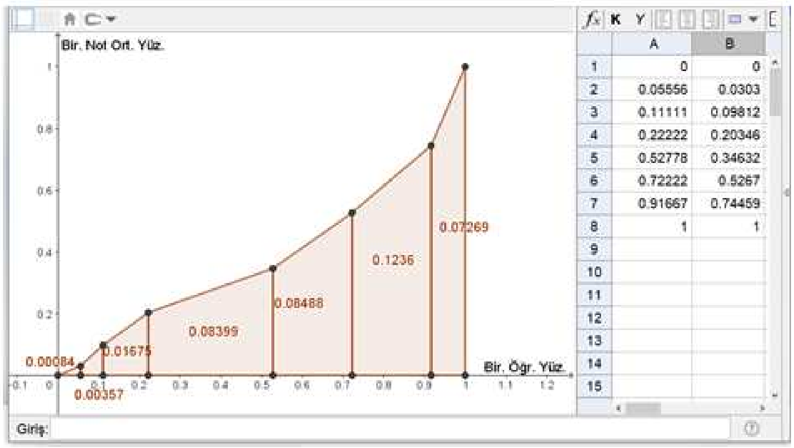

Fig. 2: Trapezoidal areas below the curve(A), A area (0,5- B);B=0,5-0,1136447=0,3863553 Gini=A/A+B = 0,1136447/(0,1136447+0,3863553)=0,227352894

The Gini coefficient of 0.22 is that the elementary mathematics teachers' academic achievement is distributed to students fairly or 36 students share the achievement fairly. If the Gini coefficient is 0.45, it is suggested that the prospective mathematics teacher candidates did not distribute their academic achievement fairly in the courses of Statistics and Probability or 16 students could not share the achievement fairly, but they fit the expected situation in the other years, and further evaluations can be made by following the success of other courses in those years.

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Fractional Solutions of a k-hypergeometric Differential Equation

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Abstract: In the present work, we study the second order homogeneous k-hypergeometric differential equation by utilizing the discrete fractional Nabla calculus operator. As a result, we obtained a novel exact fractional solution to the given equation.

Keywords: Discrete fractional, the k-hypergeometric differential equation, Nabla operator.

1 Introduction

Fractional calculus deal with derivatives and integrals of arbitrary orders, their applications seem in different areas of science such as physics, applied mathematics, chemistry, engineering [1–4]. Mathematical models have significant applications in physical and technical processing phenomena [5–9]. The solutions of the differential equations relevant to many interesting special functions in mathematics, physics, and engineering, such as the hypergeometric series [10], the zeta function [11], the continued fraction [12], the power series [13], the Fourier analysis [14]. The discrete fractional Nabla calculus operator have been applied to various singular ordinary equations such as the second-order linear ordinary differential equation of hypergeometric type [15], the modified Bessel differential equation [16], the radial equation of the fractional Schrödinger equation [17, 18], the Gauss equation [19], the non-Fuchsian differential equation [20], the Chebyshev's equation [21]. The aim of this study is to apply the Nabla calculus operator to a well-known ordinary differential equation k-hypergeometric equation [22], which is expressed by

$$kr\left(1-kr\right)\frac{d^2w}{dr^2}+\left[\alpha-\left(k+\rho+\sigma\right)kr\right]\frac{dw}{dr}-\rho\sigma w=v\left(r\right), \tag{1}$$

where $k\in\mathbb{R}^+, \alpha,\ \rho,\ \sigma\in\mathbb{R}^+$and $v\left(r\right)$ is holomorphic in an interval $D\subseteq\mathbb{C}$. If $k=1$ and the function $v\left(r\right)$ be vanishes identically, then Eq. (1) reduce to a linear homogenous hypergeometric ordinary differential equation (ODE) as follows

$$r\left(1-r\right)\frac{d^2w}{dr^2}+\left[\alpha-\left(1+\rho+\sigma\right)r\right]\frac{dw}{dr}-\rho\sigma w=0. \tag{2}$$

Many researchers have been studied the hypergeometric differential equation by different schemes, such as Kummer, presented the concurrent of hypergeometric equation in physical models [23]. Campos, finalize that this kind of equation contains complex calculations, and also the singularities of the differential equation are orderly. [24].

2 Preliminaries

Here, we have some imperative knowledge about the discrete fractional calculus theory and also some necessary notes, $\mathbb{N}$ is the set of natural numbers including zero, and $\mathbb{Z}$ is the set of integers. The $\mathbb{N}_b=\{b,b+1,b+2,...\}$ for $b\in\mathbb{Z}$. Let $f(t)$ and $g(t)$ are the real valued functions defined on $\mathbb{N}_0^+$. For more details see [15–21].
Definition 1. The rising factorial power is defined by

$$z^{\bar{n}}=t\left(z+1\right)\left(z+2\right)...\left(z+n-1\right),\ n\in\mathbb{N},\ z^{\bar{0}}=1.$$

Given α be a real number, then $z^{\bar{\alpha}}$ is expressed by

$$t^{\bar{\alpha}}=\frac{\Gamma\left(t+\alpha\right)}{\Gamma\left(t\right)}, \tag{3}$$

where $z\in\mathbb{R}\backslash\{...,-2,-1,0\}$, and $0^{\bar{\alpha}}=0$.
Let us symbolize that

$$\nabla\left(z^{\overline{\alpha}}\right)=\alpha z^{\overline{\alpha-1}}, \tag{4}$$

here $\nabla u(z) = u(z) - u(z-1)$. For $n = 2, 3, \ldots$ describe ∇^n by $\nabla^n = \nabla\nabla^{n-1}$.
Definition 2. The α^{th} order fractional sum of f is defined by

$$\nabla_b^{-\alpha} f(z) = \sum_{s=b}^{z} \frac{[s-\delta(z)]^{\overline{\alpha-1}}}{\Gamma(\alpha)} f(s), \tag{5}$$

where $z \in \mathbb{N}_b,\ \delta(z) = z-1$ is backward jump operator.
Theorem 1. Let $f(z)$ and $g(z): \mathbb{N}_0^+ \to \mathbb{R},\ \alpha,\ \beta > 0$, and $h,\ v$ are constants, then

$$\nabla^{-\alpha}\nabla^{-\beta} f(z) = \nabla^{-(\alpha+\beta)} f(z) = \nabla^{-\beta}\nabla^{-\alpha} f(z) \tag{6}$$

$$\nabla^{\alpha}[hf(z) + vg(z)] = h\nabla^{\alpha} f(z) + v\nabla^{\alpha} g(z) \tag{7}$$

$$\nabla\nabla^{-\alpha} f(z) = \nabla^{-(\alpha-1)} f(z) \tag{8}$$

$$\nabla^{-\alpha}\nabla f(z) = \nabla^{(1-\alpha)} f(z) - \begin{pmatrix} z+\alpha-2 \\ z-1 \end{pmatrix} f(0) \tag{9}$$

Lemma 1. For all $\alpha > 0,\ \alpha^{th}$ order fractional difference of the product fg is expressed by

$$\nabla_0^{\alpha}(fg)(z) = \sum_{n=0}^{z} \begin{pmatrix} \alpha \\ n \end{pmatrix} \left[\nabla_0^{\alpha-n} f(z-n)\right] \left[\nabla^n g(z)\right]. \tag{10}$$

Lemma 2. If the function $f(t)$ is single valued and analytic, then

$$[f_\alpha(z)]_\beta = f_{\alpha+\beta}(z) = \left[f_\beta(z)\right]_\alpha,\ \left[f_\alpha(z) \neq 0,\ f_\beta(z) \neq 0,\ \alpha,\ \beta \in \mathbb{R},\ z \in \mathbb{N}\right]. \tag{11}$$

3 Main results

Theorem 2. Let $w \in \{w : 0 \neq |w_\vartheta| < \infty,\ \vartheta \in \mathbb{R}\}$, and then the homogeneous k-hypergeometric equation is given by

$$w_2 kr(1-kr) + w_1[\alpha - (k+\rho+\sigma)kr] - w\rho\sigma = 0, \tag{12}$$

has a particular solution of the form

$$w = h\left\{(r)^{-\left(\frac{1}{k}(\vartheta\theta k+\alpha)\right)}(1-kr)^{-\left(\frac{1}{k}(\vartheta\theta k+\rho+\sigma-\alpha+k)\right)}\right\}_{-(\vartheta+1)},\ r \neq \left\{0, \frac{1}{k}\right\}. \tag{13}$$

where $w_m(r) = \frac{d^m w}{dr^m},\ (m = 0, 1, 2),\ w_0 = w(r)$, and $\alpha,\ \rho,\ \sigma$ are given constants as well as h is a constant of integration.
Proof. When we applied the discrete fractional calculus operator to both sides of Eq. (12), we have

$$\nabla^{\vartheta} w_2 kr(1-kr) + \nabla^{\vartheta} w_1[\alpha - (k+\rho+\sigma)kr] - \nabla^{\vartheta}(w\rho\sigma) = 0, \tag{14}$$

using Eq. (8), and Eq. (9) together with Eq. (14), one may obtain

$$\begin{aligned} &w_{\vartheta+2} kr(1-kr) + w_{\vartheta+1}[\vartheta\theta k(1-2kr) + \alpha - (k+\rho+\sigma)kr] \\ &+ w_\vartheta\left[-\vartheta(\vartheta-1)\theta^2 k^2 + \vartheta\theta(-(k+\rho+\sigma)k) - \rho\sigma\right] = 0, \end{aligned} \tag{15}$$

where θ is a shift operator.
We choose ϑ such that

$$\vartheta(\vartheta-1)\theta^2 k^2 + \vartheta\theta\left(k^2 + k\rho + k\sigma\right) + \rho\sigma = 0,$$

$$\vartheta = \frac{\left[\theta k - (k+\rho+\sigma) \pm \sqrt{((k+\rho+\sigma)-\theta k)^2 - 4\rho\sigma}\right]}{2\theta k}, \tag{16}$$

and let $(k+\rho+\sigma-\theta k)^2 \geq 4\rho\sigma$, then we have

$$w_{\vartheta+2} kr (1-kr) + w_{\vartheta+1} [\vartheta\theta k (1-2kr) + \alpha - (k+\rho+\sigma) kr] = 0, \tag{17}$$

and set

$$w_{\vartheta+1} = W = W(r), \ \left(w = W_{-(\vartheta+1)}\right). \tag{18}$$

Therefore

$$W_1 + W \left[\frac{\vartheta\theta k (1-2kr) + \alpha - (k+\rho+\sigma) kr}{kr(1-kr)}\right] = 0, \tag{19}$$

by using Eq. (17), and Eq. (18), then the solution of the ODE Eq. (19) has the form

$$W = h(r)^{-\left(\frac{1}{k}(\vartheta\theta k+\alpha)\right)} (1-kr)^{-\left(\frac{1}{k}(\vartheta\theta k+\rho+\sigma-\alpha+k)\right)}. \tag{20}$$

4 Conclusion

In the present study, we applied the discrete fractional Nabla calculus operator to the homogeneous k-hypergeometric differential equation. As a result, we obtained a new exact discrete fractional solution.

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The Space bv_k^θ and Matrix Transformations

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Abstract: In this study, we introduce the space bv_k^θ, give its some algebraic and topological properties, and also characterize some matrix operators defined on that space. Also we extend some well known results.

Keywords: BK spaces, Matrix transformations, Sequence spaces.

1 Introduction

Let ω be the set of all complex sequences, ℓ_k and c be the sets of k-absolutely convergent series and convergent sequences, respectively. By bv we denote the space of all sequences of bounded variation, i.e.,

$$bv = \{x \in w : \Delta x \in \ell_k\}.$$

Let U and V be subspaces of w and $A = (a_{nv})$ be an arbitrary infinite matrix of complex numbers. By $A(x) = (A_n(x))$, we denote the A-transform of the sequence $x = (x_v)$, i.e.,

$$A_n(x) = \sum_{v=0}^{\infty} a_{nv} x_v,$$

provided that the series is convergent for $n \geq 0$. Then, we say that A defines a matrix transformation from U into V, and denote it by $A \in (U, V)$ if the sequence $A(x) = (A_n(x)) \in V$ for every sequence $x \in U$, also the sets $U^\beta = \{\varepsilon = (\varepsilon_v) : \Sigma \varepsilon_v x_v \text{ converges for all } x \in U\}$ and

$$U_A = \{x \in \omega : A(x) \in U\} \tag{1}$$

are called the β dual of U and the domain of a matrix A in U. Further, $U \subset w$ is said to be a BK-space if it is a Banach space with continuous coordinates $p_n : U \to \mathbb{C}$ defined by $p_n(x) = x_n$ for $n \geq 0$. The sequence (e_v) is called a Schauder base (or briefly base) for a normed sequence space U if for each $x \in U$ there exist unique scalar coefficients (x_v) such that

$$\lim_{m\to\infty} \left\| x - \sum_{v=0}^{m} x_v e_v \right\| = 0,$$

and we write

$$x = \sum_{v=0}^{\infty} x_v e_v.$$

An infinite matrix $A = (a_{nv})$ is called a triangle if $a_{nn} \neq 0$ and $a_{nv} = 0$ for all $v > n$ for all n, v [1].
We define the notations Γ_c, Γ_∞ and Γ_s for $v = 1, 2, ...$, as follows:

$$\Gamma_c = \left\{ \varepsilon = (\varepsilon_v) : \lim_m \sum_{v=r}^{m} \varepsilon_v \text{ exists for } r = 1, 2, ... \right\},$$

$$\Gamma_\infty = \left\{ \varepsilon = (\varepsilon_v) : \sup_{m,r} \left| \sum_{v=r}^{m} \varepsilon_v \right| < \infty,\ r = 1, 2, ... \right\},$$

and

$$\Gamma_s = \left\{ \varepsilon = (\varepsilon_v) : \sup_m \sum_{r=1}^{m} \left| \theta_r^{-1/k^*} \sum_{v=r}^{m} \varepsilon_v \right|^{k^*} < \infty \right\},$$

where k^* is the conjugate of k, that is, $1/k + 1/k^* = 1$, and $1/k^* = 0$ for $k = 1$.

More recently some new sequence spaces by means of the matrix domain of a particular limitation method or absolute summability methods have been defined and studied by several authors in many research papers (see, for instance [2–8]). In this study, we introduce the space bv_k^θ, give its some algebraic and topological properties and characterize some matrix operators defined on that space. Also we extend some well known results.

The following lemmas are needed in proving our theorems.

Lemma 1. Let $1 \leq k < \infty$. Then, $A \in (\ell, \ell_k)$ if and only if

$$\sup_v \sum_{n=0}^{\infty} |a_{nv}|^k < \infty,$$

[9].

Lemma 2.
a-)

$$A \in (\ell, c) \Leftrightarrow (i) \;\; \lim_n a_{nv} \text{ exists for each } v, \text{ and } \;\; (ii) \;\; \sup_{n,v} |a_{nv}| < \infty.$$

b-) Let $1 < k < \infty$.Then $A \in (\ell_k, c) \Leftrightarrow (i)$ holds and

$$\sup_n \sum_{v=0}^{\infty} |a_{nv}|^{k^*} < \infty$$

[10].

2 The space bv_k^θ and matrix operators

In this section we introduce the space bv_k^θ as

$$bv_k^\theta = \left\{ x = (x_k) \in w : \left(\theta_n^{1/k^*} \triangle x_n \right) \in \ell_k \right\},$$

where (θ_n) is a sequence of nonnegative terms, $1 \leq k < \infty$ and $\triangle x_n = x_n - x_{n-1}$ for all n. Note that it includes some known spaces. For example, it is reduced to bv^k for $\theta_n = 1$ for all n and $bv_1^\theta = bv$, which have been studied by Malkowsky et al [11] and Jarrah and Malkowsky [6]. Moreover, recently, Başar et al [3] have defined the sequence space $bv\,(u, p)$ and proved that this space is linearly isomorphic to the space $\ell(p)$ of Maddox [12] as generalized to paranormed space.

It is redefined as $bv_k^\theta = (\ell_k)_A$ with the notation (1) , where the matrix A is defined by

$$a_{nv} = \begin{cases} -\theta_n^{1/k^*}, & v = n-1, \\ \theta_n^{1/k^*}, & v = n, \\ 0, & v \neq n, n-1. \end{cases}$$

Further, $\left| N_p^\theta \right|_k = \left(bv_k^\theta \right)_A$ and $|C_\alpha|_k = \left(bv_k^\theta \right)_B$ where A and B are Cesàro and Nörlund means of series Σx_n (see [8],[5, 13]).

Now we begin with topological properties of bv_k^θ, which also can be deduced from [3] .

Lemma 3. Let $1 \leq k < \infty$ and (θ_n) be a sequence of nonnegative numbers. Then,
a-) The space bv_k^θ is a BK-space and norm isomorphic to the space ℓ_k, $i.e.$, $bv_k^\theta \approx \ell_k$.
b-) $\left(bv_k^\theta \right)^\beta = \Gamma_c \cap \Gamma_s$ for $1 < k < \infty$ and $(bv)^\beta = \Gamma_c \cap \Gamma_\infty$ for $k = 1$.
c-) Define the sequence $b^{(j)} = \left(b_n^{(j)} \right)$ such that, for $j, n \geq 0$,

$$b_n^{(j)} = \begin{cases} \theta_j^{-1/k^*}, & n \geq j, \\ 0, & n < j. \end{cases}$$

Then, the sequence $b^{(j)} = \left(b_n^{(j)} \right)$ is the base of bv_k^θ.

Proof: a-) Since ℓ_k is a BK-space with respect to its usual norm and A is a triangle matrix, Theorem 4.3.2 of Wilansky [1, p. 61] gives the fact that bv_k^θ is a BK-space for $1 \leq k < \infty$. Now, consider $T : bv_k^\theta \to \ell_k$ defined by $y = T(x) = \left(\theta_n^{1/k^*} \Delta x_n \right)$ for all $x \in bv_k^\theta$. Then, it is clear that T is a linear operator, and surjective since, if $y = (y_n) \in \ell_k$, then $x = (x_n) = \left(\Sigma_{j=0}^n \theta_j^{-1/k^*} y_j \right) \in bv_k^\theta$, and also one to one. Further, it preserves the norm, since

$$\|T(x)\|_{\ell_k} = \left(\sum_{n=0}^{\infty} \theta_n^{k-1} |\triangle x_n|^k \right)^{1/k} = \|x\|_{bv_k^\theta},$$

which completes the proof.

b-) This part can be proved together with Lemma 2.

c-) Since the sequence $e^{(j)}$ is a base of ℓ_k, where $e^{(j)} = \left(e_n^{(j)}\right)_{n=0}^{\infty}$ is the sequence whose only non-zero term is 1 in the nth place for each $n \in \mathbb{N}$, it is clear that the sequence $b^{(j)}$ is the base of bv_k^{θ}. In fact, we first note that $T^{-1}(e^{(j)}) = b^{(j)}$. Now, if $x \in bv_k^{\theta}$, then there exists $y \in \ell_k$ such that $y = T(x)$, and so it follows from (a) that

$$\left\| x - \sum_{j=0}^{m} x_j b^{(j)} \right\|_{bv_k^{\theta}} = \left\| y - \sum_{j=0}^{m} y_j e^{(j)} \right\|_{\ell_k} \to 0 \text{ as } m \to \infty,$$

and it is easy to see that the representation $x = \sum_{j=0}^{\infty} x_j b^{(j)}$ is unique. □

Theorem 1. *Let $A = (a_{nv})$ be an infinite matrix of complex numbers for all $n, v \geq 0$, (θ_n) be a sequence of nonnegative numbers and $1 \leq k < \infty$. Then, $A \in \left(bv, bv_k^{\theta}\right)$ if and only if*

$$\lim_{n\to\infty} \sum_{j=\nu}^{\infty} a_{nj} \text{ exists for each } v, \tag{2}$$

$$\sup_{n,v} \left| \sum_{j=v}^{\infty} a_{nj} \right| < \infty \tag{3}$$

and

$$\sup_{\nu} \sum_{n=0}^{\infty} \left| \theta_n^{1/k^*} \sum_{j=\nu}^{\infty} \left(a_{nj} - a_{n-1,j}\right) \right|^k < \infty. \tag{4}$$

Proof: $A \in \left(bv, bv_k^{\theta}\right)$ iff $\left(a_{nj}\right)_{j=0}^{\infty} \in bv^{\beta}$ and $A\left(x\right) \in bv_k^{\theta}$ for every $x \in bv$, and also, by Lemma 3, $\left(a_{nj}\right)_{j=0}^{\infty} \in bv^{\beta}$ iff (2) and (3) hold. Now, to prove necessity and sufficiency of the condition (4), consider the operators $B : bv \to \ell$ and $B' : bv_k^{\theta} \to \ell_k$ defined by

$$B_n\left(x\right) = \Delta x_n,\ B'_n(x) = \theta_n^{1/k^*} \Delta x_n,$$

respectively. As in Lemma 3, these operators are bijection and the matrices corresponding to these operators are triangles. Further, let $x \in bv$ be given. Then, $B\left(x\right) = y \in \ell$ iff $x = S\left(y\right)$, where S is the inverse of B and it is given by

$$s_{n\nu} = \begin{cases} 1, & 0 \leq \nu \leq n, \\ 0, & \nu > n. \end{cases}$$

On the other hand, if any matrix $R = (r_{nv}) \in (\ell, c)$, then, the series $R_n(x) = \Sigma r_{nv} x_v$ is convergent uniformly in n, since, by Lemma 2, the remaining term tends to zero uniformly in n, that is,

$$\left| \sum_{v=m}^{\infty} r_{nv} x_v \right| \leq \left(\sup_{n,v} |r_{nv}| \right) \sum_{v=m}^{\infty} |x_v| \to 0 \text{ as } m \to \infty,$$

and so

$$\lim_n R_n(x) = \sum_{v=0}^{\infty} \lim_n r_{nv} x_v. \tag{5}$$

Now, it is easily seen from (2) and (3) that $H = \left(h_{mr}^{(n)}\right) \in (\ell, c)$, which gives us, by (5) , that

$$A_n(x) = \lim_m \sum_{r=0}^{m} h_{mr}^{(n)} y_r = \sum_{r=0}^{\infty} \left(\sum_{v=r}^{\infty} a_{nv} \right) y_r,$$

converges for all $n \geq 0$,where, for $r, m = 0, 1, ...,$

$$h_{mr}^{(n)} = \begin{cases} \sum_{v=r}^{m} a_{nv} s_{vr}, 0 \leq r \leq m, \\ 0, r > m. \end{cases}$$

This shows that the mapping sequence $A(x) = (A_n(x))$ exists. On the other hand, since S is the infinite triangle matrix, it is clear that $A(x) = A\left(S(y)\right) \in bv_k^{\theta}$ for every $x \in bv$ iff $B'\left(A\left(S(y)\right)\right) \in \ell_k$, i.e., $\left(B' o A o S\right)(y) \in \ell_k$, which implies that $D = B' o A o S : \ell \to \ell_k$.

Therefore, it can be written that $A : bv \to bv_k^\theta$ iff $D : \ell \to \ell_k$, and also $D = B' o \widehat{A}$, where $\widehat{A} = AoS$. Now, a few calculations reveal that

$$\widehat{a}_{nv} = \sum_{j=v}^{\infty} a_{nj} s_{jv} = \sum_{j=v}^{\infty} a_{nj}$$

and so

$$d_{nv} = \sum_{j=0}^{n} b'_{nj} \widehat{a}_{jv} = \theta_n^{1/k^*} \sum_{j=\nu}^{\infty} \left(a_{nj} - a_{n-1,j}\right)$$

Now, let us apply Lemma 1 with the matrix D. Then, it can be easily obtained from the definition of the matrix D that $D : \ell \to \ell_k$ iff condition (4) holds. This completes the proof.

□

If A is an infinite triangle matrix in Theorem 1, then (2) and (3) hold, and so it reduces to the following result.

Corollary 1. If A is an infinite triangle matrix of complex numbers for all $n, v \geq 0$ and $1 \leq k < \infty$, then, $A \in \left(bv, bv_k^\theta\right)$ if and only if

$$\sup_{\nu} \sum_{n=0}^{\infty} \left| \theta_n^{1/k^*} \sum_{j=\nu}^{n} \left(a_{nj} - a_{n-1,j}\right) \right|^k < \infty.$$

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On The Directional Associated Curves of Timelike Space Curve

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Abstract: In this work, the directional associated curves of timelike space curve in Minkowski 3-space by using q-frame are studied. We investigate quasi normal-binormal direction and donor curves of the timelike curve with q-frame. Finally, some new associated curves are constructed and plotted.

Keywords: Associated curves, Minkowski space, q-frame.

1 Introduction

The theory of curves is the one of the most important subject in differential geometry. The curves are represented in parametrized form and then their geometric properties and various quantities associated with them, such as curvature and arc length expressed via derivatives and integrals using the idea of vector calculus. There are special curves which are classical differential geometric objects. These curves are obtained by assuming a special property on the original regular curve. Some of them are Smarandache curves, curves of constant breadth, Bertrand curves, and Mannheim curves, associated curves, etc. Studying curves can be differed according to frame used for curve [1], [2], [3]. There are many studies on these special curves; for example, Choi and Kim in 2012 introduced the notion of the principal (binormal)-direction curve and principal (binormal)-donor curve of a Frenet curve and gave the relationship of curvature and torsion of its mates in both Euclidean and Minkowski spaces [4]-[5]. Also Macit and Duldul in 2014 worked on the new associated curves in $\mathbf{E^3}$ and $\mathbf{E^4}$ [6]. New associated curves by using the Bishop frame are obtained by some researches in [7], [8], [9] and [10]. In this paper, we give another approach to directional associated curves of timelike space curve with q-frame used in [11], [12], [13] and [14].

The aim of this study in this paper is to define n_q, b_q-direction curves and n_q, b_q-donor curves of timelike curve γ via the q-frame in $\mathbb{E}_1^3$ and give the relationship between q-curvatures and curvature and torsion of its mates in Minkowski space.

2 Preliminaries

Let $\alpha(t)$ be a space curve with a non-vanishing second derivative. The Frenet frame is defined as follows,

$$\mathbf{t} = \frac{\alpha'}{\|\alpha'\|}, \mathbf{b} = \frac{\alpha' \wedge \alpha''}{\|\alpha' \wedge \alpha''\|}, \mathbf{n} = \mathbf{b} \wedge \mathbf{t}. \tag{1}$$

The curvature κ and the torsion τ are given by

$$\kappa = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3}, \tau = \frac{\det(\alpha', \alpha'', \alpha''')}{\|\alpha' \wedge \alpha''\|^2}. \tag{2}$$

The well-known Frenet formulas are given by

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}' \\ \mathbf{b}' \end{bmatrix} = v \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix}, \tag{3}$$

where

$$v = \|\alpha'(t)\|. \tag{4}$$

In order to construct the 3D curve offset, Coquillart in [15] introduced the quasi-normal vector of a space curve. The quasi-normal vector is defined for each point of the curve, and lies in the plane perpendicular to the tangent of the curve at this point.

As an alternative to the Frenet frame, a new adapted frame called q-frame in both Euclidean and Minkowski space is defined by Ekici et all in [11] and [13]. Given a space curve $\alpha(t)$ the q-frame consists of three orthonormal vectors, the unit tangent vector $\mathbf{t}$, the quasi-normal vector $\mathbf{n}_q$ and the quasi-binormal vector $\mathbf{b}_q$. The q-frame $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q, \mathbf{k}\}$ is given by

$$\mathbf{t} = \frac{\alpha'}{\|\alpha'\|}, \mathbf{n}_q = \frac{\mathbf{t} \wedge \mathbf{k}}{\|\mathbf{t} \wedge \mathbf{k}\|}, \mathbf{b}_q = \mathbf{t} \wedge \mathbf{n}_q \tag{5}$$

where $\mathbf{k}$ is the projection vector, which can be chosen $\mathbf{k} = (0, 1, 0)$ or $\mathbf{k} = (1, 0, 0)$ or $\mathbf{k} = (0, 0, 1)$.

A q-frame along a space curve is shown in Figure 1.

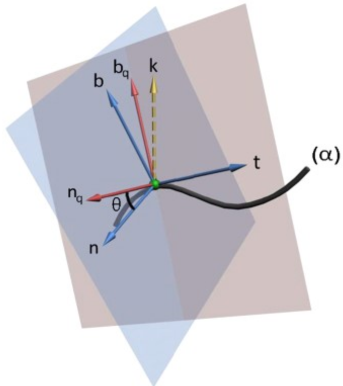

Fig. 1: The q-frame and Frenet frame

Since the derivation formula for the q-frame for the timelike curve in Minkowski space does not depend on projection vector being timelike or spacelike, we work on spacelike projection vector without loss of generality.

In [12], the variation equations of the directional q-frame for the timelike space curve when tangent vector (timelike), projection vector $\mathbf{k} = (0, 1, 0)$ (spacelike), quasi-normal vector (spacelike) and quasi-binormal vector (spacelike) are given by

$$\begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix} = \begin{bmatrix} 0 & k_1 & k_2 \\ k_1 & 0 & k_3 \\ k_2 & -k_3 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n}_q \\ \mathbf{b}_q \end{bmatrix} \tag{6}$$

where the q-curvatures are

$$k_1 = \langle \mathbf{t}', \mathbf{n}_q \rangle, \ \ k_2 = \langle \mathbf{t}', \mathbf{b}_q \rangle, \ \ k_3 = \langle \mathbf{n}_q', \mathbf{b}_q \rangle,$$

In the three dimensional Minkowski space $\mathbb{R}^3_1$, the inner product and the cross product of two vectors $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{v} = (v_1, v_2, v_3) \in \mathbb{R}^3_1$ are defined as

$$< \mathbf{u}, \mathbf{v} >= u_1 v_1 + u_2 v_2 - u_3 v_3 \tag{7}$$

and

$$\mathbf{u} \wedge \mathbf{v} = (u_3 v_2 - u_2 v_3, u_1 v_3 - u_3 v_1, u_1 v_2 - u_2 v_1) \tag{8}$$

where $e_1 \wedge e_2 = e_3$, $e_2 \wedge e_3 = -e_1$, $e_3 \wedge e_1 = -e_2$, respectively [16].

The norm of the vector $\mathbf{u}$ is given by

$$\|\mathbf{u}\| = \sqrt{|\langle u, u \rangle|}. \tag{9}$$

We say that a Lorentzian vector $\mathbf{u}$ is spacelike, lightlike or timelike if $\langle \mathbf{u}, \mathbf{u} \rangle > 0$, $\langle \mathbf{u}, \mathbf{u} \rangle = 0$ and $\mathbf{u} \neq 0$, $\langle \mathbf{u}, \mathbf{u} \rangle < 0$, respectively. In particular, the vector $\mathbf{u} = 0$ is spacelike.

An arbitrary curve $\alpha(s)$ in $\mathbb{R}^3_1$ can locally be spacelike, timelike or null(lightlike), if all its velocity vectors $\alpha'(s)$ are respectively spacelike, timelike or null.

A null curve α is parameterized by pseudo-arc s if $\langle \alpha''(s), \alpha''(s) \rangle = 1$. On the other hand, a non-null curve α is parameterized by arc-lenght parameter s if $\langle \alpha'(s), \alpha'(s) \rangle = \pm 1$ [17] and [18].

Then Frenet formulas of timelike curve may be written as

$$\frac{d}{dt} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = v \begin{bmatrix} 0 & \kappa & 0 \\ \kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} \tag{10}$$

where $v = \|\alpha'(t)\|$. The Minkowski curvature and torsion of timelike curve $\alpha(t)$ are obtained by

$$\kappa =< \mathbf{t}', \mathbf{n} >, \ \ \tau =< \mathbf{n}', \mathbf{b} >,$$

respectively [16] and [19].

Let x and y be future painting (or post painting) timelike vectors in E_1^3 , then there is an unique real number $\theta \geq 0$ such that

$$\langle x, y\rangle = \|x\| \, \|y\| \cosh\theta.$$

This number is called the hyperbolic angle between the vectors x and y [19]. Let x and y be spacelike vectors in E_1^3 that span spacelike vector subspace. Then, there is an unique real number $\theta \geq 0$ such that

$$\langle x, y\rangle = \|x\| \, \|y\| \cos\theta.$$

This number is called the spacelike angle between the vectors x and y.

Let x be a spacelike and y be a timelike vectors in E_1^3 , then there is an unique real number $\theta \geq 0$ such that

$$\langle x, y\rangle = \|x\| \, \|y\| \sinh\theta.$$

This number is called the timelike angle between the vectors x and y [19]. The relation between Frenet ($\mathbf{n}$ is timelike) and q-frame ($\mathbf{t}$ is timelike) is given as

$$\begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sinh\theta & \cosh\theta \\ 0 & \cosh\theta & \sinh\theta \end{bmatrix} . \begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix}, \tag{11}$$

where the angle is between $\mathbf{n}$ and $\mathbf{n}_q$.

Also the relation between q-curvatures and curvature and torsion are

$$k_1 = \kappa \sinh\theta, \;\; k_2 = \kappa\cosh\theta, \;\; k_3 = -d\theta + \tau. \tag{12}$$

The relation between Frenet ($\mathbf{b}$ is timelike) and q-frame ($\mathbf{t}$ is timelike) is given as

$$\begin{bmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cosh\theta & \sinh\theta \\ 0 & -\sinh\theta & -\cosh\theta \end{bmatrix} . \begin{bmatrix} \mathbf{t}' \\ \mathbf{n}_q' \\ \mathbf{b}_q' \end{bmatrix}, \tag{13}$$

where the angle is between $\mathbf{b}$ and $\mathbf{n}_q$.

Also the relation between q-curvatures and curvature and torsion are

$$k_1 = \kappa \cosh\theta, \;\; k_2 = \kappa\sinh\theta, \;\; k_3 = -d\theta - \tau. \tag{14}$$

3 Directional Associated Curves of Timelike Space Curve

In this section, we inverstigate $\mathbf{n}_q$ and $\mathbf{b}_q-$ direction and donor curves of the timelike curve with q-frame in $\mathbb{E}_1^3$. For a Frenet frame $\gamma : I \rightarrow \mathbb{E}_1^3$, consider a vector field V with q frame as follows:

$$V(s) = u(s)t(s) + v(s)n_q(s) + w(s)b_q(s), \tag{15}$$

where $u, v,$ and w are functions on I satisfying

$$u^2(s) + v^2(s) - w^2(s) = 1. \tag{16}$$

Then, an integral curve $\overline{\gamma}(s)$, that is $V(\overline{\gamma}(s)) = \overline{\gamma}'(s)$, of V defined on I is a unit speed curve in $\mathbb{E}_1^3$.

Let γ be a timelike curve in $\mathbb{E}_1^3$. An integral curve of n_q is called n_q-direction curve of the timelike curve γ via q-frame.

Remark 1. *A n_q-direction curve is an integral curve of the equation (15) with* $u(s) = w(s) = 0, v(s) = 1.$

Let γ be a timelike curve in $\mathbb{E}_1^3$. An integral curve of b_q is called b_q-direction curve of the timelike curve γ via q-frame.

Remark 2. *A b_q-direction curve is an integral curve of the equation (15) with* $u(s) = v(s) = 0, w(s) = 1.$

3.1 $\mathbf{n}_q-$ *direction and donor curves of the timelike curve with q-frame*

Theorem 1. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the q-curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the n_q-direction curve of γ with the q-curvature $\overline{k}_1, \overline{k}_2, \overline{k}_3$. Then we have*

$$\begin{gathered} \overline{t} = n_q, \;\; \overline{n}_q = -t, \;\; \overline{b}_q = b_q \\ \overline{k}_1 = |k_1| \;\; or \;\; \overline{k}_1 = \sqrt{|2k_3^2 - k_1^2|}, \;\; \overline{k}_2 = k_3, \;\; \overline{k}_3 = k_2. \end{gathered} \tag{17}$$

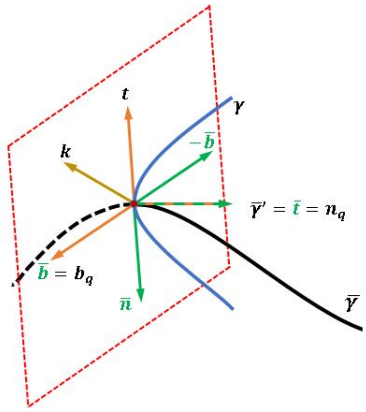

Fig. 2: n_q direction curve

Proof. By definition of n_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = n_q. \tag{18}$$

Geometrically, since $\overline{n}_q$ and t lie on the same plane, we can take $\overline{n}_q = -t$. The vectorial product of $\overline{t}$ and $\overline{n}_q$ is as follows:

$$\overline{b}_q = \overline{n}_q \times \overline{t} \tag{19}$$

therefore, $\overline{b}_q = b_q$. Differentiating the expression (18) and then taking its norm, we find

$$\overline{k}_1 = |k_1| \;\text{ or }\; \overline{k}_1 = \sqrt{|2k_3^2 - k_1^2|}. \tag{20}$$

Using definition of $q-$ curvatures and derivation formula of $q-$ frame, one can get $\overline{k}_2 = k_3$, and $\overline{k}_3 = k_2$.

Theorem 2. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the q−curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the n_q-direction curve of the timelike curve γ with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$. Then we have*

$$\begin{aligned} \overline{t} = n_q, \quad \overline{n} = -t, \quad \overline{b} = b_q \\ \overline{\kappa} = \sqrt{|-k_1^2 + k_3^2|}, \quad \overline{\tau} = -k_2. \end{aligned} \tag{21}$$

Proof. By definition of n_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = n_q. \tag{22}$$

Differentiating the expression (22) and then taking its norm, we find

$$\overline{\kappa} = \sqrt{|-k_1^2 + k_3^2|} \tag{23}$$

Differentiation of the expressions (22) gives us

$$\overline{n} = -t. \tag{24}$$

The vectorial product of $\overline{t}$ and $\overline{n}$ is as follows:

$$\overline{b} = \overline{n} \times \overline{t}. \tag{25}$$

Using the expressions (22), (24) in (25) we find that

$$\overline{b} = b_q. \tag{26}$$

Finally, differentiating (26) and using (24) in it, we have

$$\overline{\tau} = -k_2. \tag{27}$$

Corollary 1. *Let γ be a timelike curve in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the n_q-direction curve of γ. The Frenet frame of $\overline{\gamma}$ is given in terms of the $q-$ frame as follows:*

$$\begin{aligned} &\overline{t}(s) = \overline{n}_q(s), \\ &\overline{n}(s) = -\sinh(\textstyle\int k_2(s)ds)\overline{n}_q(s) + \cosh(\int k_2(s)ds)\overline{b}_q(s), \\ &\overline{b}(s) = \cosh(\textstyle\int k_2(s)ds)\overline{n}_q(s) - \sinh(\int k_2(s)ds)\overline{b}_q(s). \end{aligned} \tag{28}$$

Proof. It is straightforwardly seen by substituting (23) and (27) into (11).

Corollary 2. *If the curve γ is a n_q-donor curve of the curve $\overline{\gamma}$ with the curvatures k_1, k_2, k_3, then the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ of the timelike curve γ are given by*

$$\overline{\tau} = \sqrt{|-k_1^2 + k_3^2|}, \quad \overline{\kappa} = \pm k_2 + \left(\frac{k_3^2}{-k_1^2 + k_3^2}\right)\left(\frac{k_1}{k_3}\right)' \tag{29}$$

Proof. Taking the squares of (23) and (27)**,** then subtracting them side by side by using (12) gives us the equation (29).

Corollary 3. *Let γ be a timelike curve with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the n_q-direction curve of γ with the curvatures k_1, k_2, k_3. Then it satisfies*

$$\frac{k_2}{k_1} = \coth\theta, \quad \frac{\overline{\tau}}{\overline{\kappa}} = \pm\frac{k_2}{\sqrt{-k_1^2 + k_3^2}} + \frac{k_3^2}{(-k_1^2 + k_3^2)^{\frac{3}{2}}}\left(\frac{k_1}{k_3}\right)' \tag{30}$$

Proof. It is straightforwardly seen by substituting the expressions (23), (27) and (29) into (12).

3.2 $\mathbf{b}_q-$ *direction and donor curves of the timelike curve with q-frame*

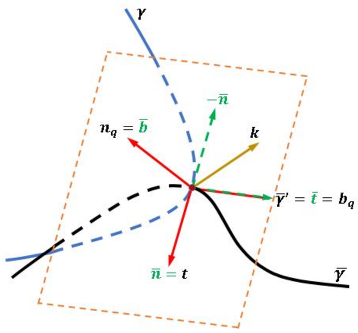

Fig. 3: b_q direction curve

Theorem 3. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the q-curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the n_q-direction curve of γ with the q-curvature $\overline{k}_1, \overline{k}_2, \overline{k}_3$. Then we have*

$$\begin{gathered} \overline{t} = n_q, \ \ \overline{n}_q = -t, \ \ \overline{b}_q = b_q \\ \overline{k}_1 = |k_1| \ \ or \ \ \overline{k}_1 = \sqrt{|2k_3^2 - k_1^2|}, \ \ \overline{k}_2 = k_3, \ \ \overline{k}_3 = k_2. \end{gathered} \tag{31}$$

Proof. By definition of n_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = n_q. \tag{32}$$

Geometrically, since $\overline{n}_q$ and t lie on the same plane, we can take $\overline{n}_q = -t$. The vectorial product of $\overline{t}$ and $\overline{n}_q$ is as follows:

$$\overline{b}_q = \overline{n}_q \times \overline{t} \tag{33}$$

therefore, $\overline{b}_q = b_q$. Differentiating the expression (32) and then taking its norm, we find

$$\overline{k}_1 = |k_1| \ \text{ or } \ \overline{k}_1 = \sqrt{|2k_3^2 - k_1^2|}. \tag{34}$$

Using definition of $q-$ curvatures and derivation formula of $q-$ frame, one can get

$$\overline{k}_2 = k_3 \ \text{ and } \ \overline{k}_3 = k_2. \tag{35}$$

Theorem 4. *Let γ be a timelike space curve in $\mathbb{E}_1^3$ with the $q-$curvatures k_1, k_2, k_3 and $\overline{\gamma}$ be the b_q-direction curve of the timelike curve γ with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$. Then we have*

$$\begin{gathered} \overline{t} = b_q, \quad \overline{n} = t, \quad \overline{b} = n_q \\ \overline{\kappa} = \sqrt{|-k_2^2 + k_3^2|}, \quad \overline{\tau} = -k_1. \end{gathered} \tag{36}$$

Proof. By definition of b_q-direction curve of γ, we can write

$$\overline{\gamma}' = \overline{t} = b_q. \tag{37}$$

Differentiating the expression (37) and then taking its norm, we find

$$\overline{\kappa} = \sqrt{|-k_2^2 + k_3^2|} \tag{38}$$

Differentiation of the expressions (37) with using of (38) gives us

$$\overline{n} = t. \tag{39}$$

The vectorial product of $\overline{t}$ and $\overline{n}$ is as follows:

$$\overline{b} = \overline{n} \times \overline{t}. \tag{40}$$

Using the expressions (37), (39) in (40) we find that

$$\overline{b} = n_q. \tag{41}$$

Finally, differentiating (41) and using definition of curvature, we have

$$\overline{\tau} = k_1 \tag{42}$$

which proves theorem.

Corollary 4. *Let γ be a timelike curve in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the b_q-direction curve of γ. The Frenet frame of $\overline{\gamma}$ is given in terms of the $q-$ frame as follows:*

$$\begin{aligned}
&\overline{t}(s) = \overline{b}_q(s), \\
&\overline{n}(s) = \cosh(\textstyle\int k_1(s)ds)\overline{n}_q(s) + \sinh(\textstyle\int k_1(s)ds)\overline{b}_q(s), \\
&\overline{b}(s) = -\sinh(\textstyle\int k_1(s)ds)\overline{n}_q(s) - \cosh(\textstyle\int k_1(s)ds)\overline{b}_q(s).
\end{aligned} \tag{43}$$

Proof. It is straightforwardly seen by substituting (38) and (42) into (13).

Corollary 5. *If the curve γ is a n_q-donor curve of the curve $\overline{\gamma}$ with the curvatures k_1, k_2, k_3, then the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ of the timelike curve γ are given by*

$$\overline{\tau} = \sqrt{|-k_2^2 + k_3^2|}, \quad \overline{\kappa} = \pm k_1 + (\frac{k_3^2}{-k_2^2 + k_3^2})(\frac{k_2}{k_3})' \tag{44}$$

Proof. Taking the squares of (38) and (42), then subtracting them side by side by using (14) gives us the equation (44).

Corollary 6. *Let γ be a timelike curve with the curvature $\overline{\kappa}$ and the torsion $\overline{\tau}$ in $\mathbb{E}_1^3$ and $\overline{\gamma}$ be the n_q-direction curve of γ with the curvatures k_1, k_2, k_3. Then it satisfies*

$$\frac{k_2}{k_1} = \tanh\theta, \quad \frac{\overline{\tau}}{\overline{\kappa}} = \pm\frac{k_1}{\sqrt{-k_2^2 + k_3^2}} + \frac{k_3^2}{(-k_2^2 + k_3^2)^{\frac{3}{2}}}(\frac{k_2}{k_3})' \tag{45}$$

Proof. It is straightforwardly seen by substituting the expressions (38), (42) and (44) into (14).

4 Examples

In this section, an example of directional associated curves of timelike space curve with q-frame are constructed and plotted.

Example 1. Consider a timelike curve

$$\gamma(t) = (-\frac{5}{9}\cosh(3t), \frac{4}{3}t, -\frac{5}{9}\sinh(3t)).$$

The Frenet frame vectors and curvatures are calculated by

$$\begin{aligned}
&\mathbf{t} = \big(-\tfrac{5}{3}\sinh(3t), \tfrac{4}{3}, -\tfrac{5}{3}\cosh(3t)\big), \\
&\mathbf{n} = \big(-\cosh(3t), 0, -\sinh(3t)\big), \\
&\mathbf{b} = \big(\tfrac{4}{3}\sinh(3t), -\tfrac{5}{3}, \tfrac{4}{3}\cosh(3t)\big), \\
&\kappa = 5, \quad \tau = 4.
\end{aligned}$$

The q-frame vectors and curvatures are obtained by

$$
\begin{aligned}
\mathbf{t} &= \big(-\tfrac{5}{3}\sinh(3t), \tfrac{4}{3}, -\tfrac{5}{3}\cosh(3t)\big),\\
\mathbf{n_q} &= \big(-\cosh(3t), 0, -\sinh(3t)\big),\\
\mathbf{b_q} &= \big(-\tfrac{4}{3}\sinh(3t), \tfrac{5}{3}, -\tfrac{4}{3}\cosh(3t)\big),\\
k_1 &= 5, \quad k_2 = 0, \quad k_3 = -4.
\end{aligned}
$$

n_q and b_q- direction curves of γ shown in Figure 4 are written as

$$
\begin{aligned}
\overline{\gamma} &= \big(-\tfrac{1}{3}\sinh(3t) + c_1, c_2, -\tfrac{1}{3}\cosh(3t) + c_3\big),\\
\overline{\overline{\gamma}} &= \big(-\tfrac{4}{9}\cosh(3t) + c_4, \tfrac{5}{3}t + c_5, -\tfrac{4}{9}\sinh(3t) + c_6\big),
\end{aligned}
$$

respectively.

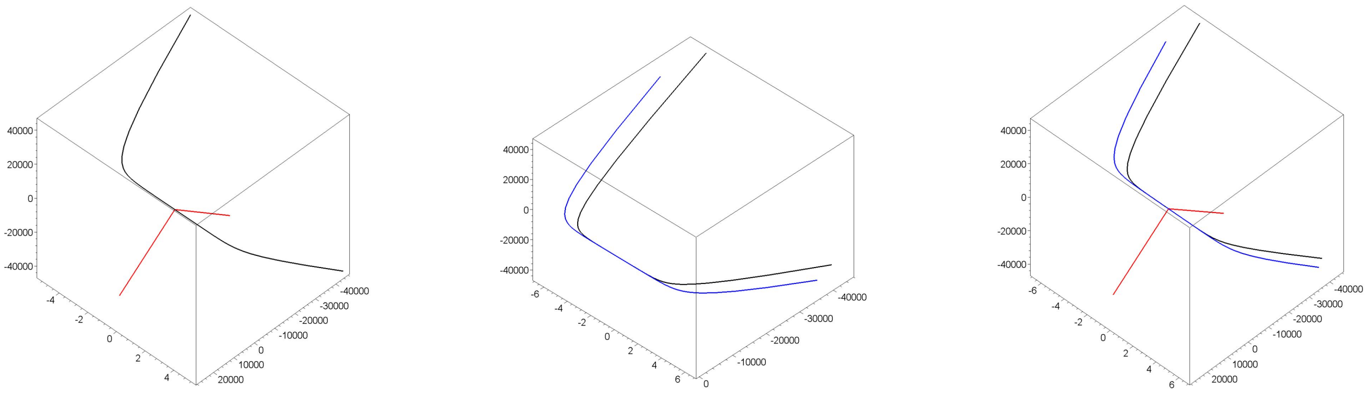

Fig. 4: Timelike curve (black), n_q direction curve (red) and b_q direction curve (blue) for $c_i = 0$.

All the figures in this study were created by using maple programme.

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De-Moivre and Euler Formulae for Dual-Hyperbolic Numbers

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Abstract: In this study, we generalize the well-known formulae of de-Moivre and Euler of hyperbolic numbers to dual-hyperbolic numbers. Furthermore, we investigate the roots and powers of a dual-hyperbolic number by using these formulae. Consequently, we give some examples to illustrate the main results in this paper.

Keywords: Dual number, Hyperbolic number.

1 Introduction

The number systems of two- dimensional numbers have taken place in literature with a multi-perspective approach. The hyperbolic numbers were first introduced by J. Cockle [1] and elaborated by I.M. Yaglom [2]. At the end of the 20th century, O. Bodnar, A. Stakhov and I.S. Tkachenko revealed a hyperbolic function class with gold ratio [3]. In recent years, there have been a great number of studies referring to hyperbolic numbers [4]-[9]. One of the most important recent studies has been given by A. Harkin and J. Harkin and generalized trigonometry including complex, hyperbolic and dual numbers were studied [10]. Any hyperbolic number (or split complex number, perplex number, double number) $z = x + j\,y$ is a pair of real numbers (x, y), which consists of the real unit $+1$ and hyperbolic (unipotent) imaginary unit j satisfying $j^2 = 1, j \neq \pm 1$. Therefore, hyperbolic numbers are elements of two-dimensional real algebra

$$H = \left\{ z = x + jy \,|x, y \in R \text{ and } j^2 = 1\,(j \neq \pm 1) \right\}$$

which is generated by 1 and j. The module of a hyperbolic number z is defined by

$$|z| = \left\{ \begin{array}{lll} \mp\sqrt{x^2 - y^2} & ; & |x| \geq |y| \\ \mp\sqrt{y^2 - x^2} & ; & |x| \leq |y| \end{array} \right.$$

and its argument is $\varphi = \operatorname{arctanh}\left(\frac{y}{x}\right)$ and represented by $\arg(z)$. Any hyperbolic number z can be given by one of the following forms;

$$\begin{array}{l} \text{a}-)\ z = r\,(\cosh\varphi + j\sinh\varphi) \\ \text{b}-)\ z = r\,(\sinh\varphi + j\cosh\varphi)\,. \end{array}$$

The hyperbolic number given in (a) and (b) is called the first and second type hyperbolic number, respectively, see figure 1.

On the other hand, the developments in the number theory present us new number systems including the dual numbers which are expressed by the real and dual parts similar to hyperbolic numbers. This idea was first introduced by W. K. Clifford to solve some algebraic problems [11]. Afterwards, E. Study presented different theorems with his studies on kinematics and line geometry [12].

A dual number is a pair of real numbers which consists of the real unit $+1$ and dual unit ε satisfying $\varepsilon^2 = 0$ for $\varepsilon \neq 0$. Therefore, the dual numbers are elements of two-dimensional real algebra

$$D = \left\{ z = x + \varepsilon y \mid x, y \in R, \varepsilon^2 = 0, \varepsilon \neq 0 \right\}$$

which is generated by $+1$ and ε.

Similar to the hyperbolic numbers, the module of a dual number z is defined by $|z| = |x + \varepsilon y| = |x| = r$ and its argument is $\theta = \frac{y}{x}$ and represented by $\arg(z)$. The set of all points which satisfy the equation $|z| = |x| = r > 0$ and which are on the dual plane are the lines $x = \pm r$ [2]. This circle is called the Galilean circle on a dual plane. Let S be a circle centered with O and M be a point on S. If d is the line OM, and α is the angle δ_{Od}, a Galilean circle can be seen in the following figure 2.

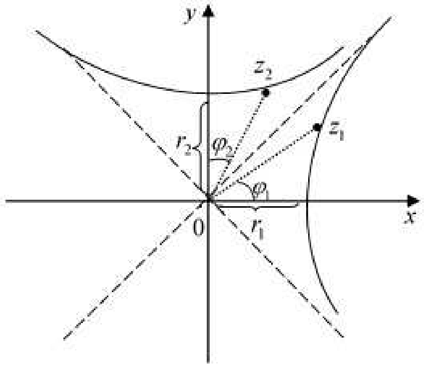

Fig. 1: Representation of hyperbolic numbers at a coordinate plane

So, one can easily see that

$$\text{cosg}\alpha = \frac{|OP|}{|OM|} = 1 \quad , \quad \text{sing}\alpha = \frac{|MP|}{|OM|} = \frac{\delta_{Od}}{1} = \alpha.$$

Moreover, the exponential representation of a dual number $z = x + \varepsilon y$ is in the form of $z = xe^{\varepsilon\alpha}$ where $\frac{y}{x}$ is dual angle and it is shown as $\arg(z) = \frac{y}{x} = \alpha$ [3]. In addition, from the definitions of Galilean cosine and sine, we realize

$$\text{cosg}(\alpha) = 1 \text{ and } \text{sing}(\alpha) = \frac{y}{x} = \alpha.$$

By considering the exponential rules, we write

$$\begin{aligned} \text{cosg}(x+y) &= \text{cosg}(x)\,\text{cosg}(y) - \varepsilon^2 \text{sing}(x)\,\text{sing}(y), \\ \text{sing}(x+y) &= \text{sing}(x)\,\text{cosg}(y) + \text{cosg}(x)\,\text{sing}(y), \\ \text{cosg}^2(x) + \varepsilon^2 \text{sing}^2(x) &= 1 \end{aligned}$$

[10].

E. Cho proved that de-Moivre formula for the hyperbolic numbers is admissible for quaternions [13]. Also, Yaylı and Kabadayı gave the de-Moivre formula for dual quaternions [14]. This formula was also investigated for the case of hyperbolic quaternions in [15]. In this study, we first introduce dual-hyperbolic numbers and algebraic expressions on dual hyperbolic numbers. We also generalize de-Moivre and Euler formulae given for hyperbolic and dual numbers to dual-hyperbolic numbers. Then we have found the roots and forces of the dual-hyperbolic numbers. Finally, the obtained results are supported by examples.

2 Dual-Hyperbolic numbers

A dual-hyperbolic number ω can be written in the form of hyperbolic pair (z_1, z_2) such that $+1$ is the real unit and ε is the dual unit. Thus, we denote dual-hyperbolic numbers set by

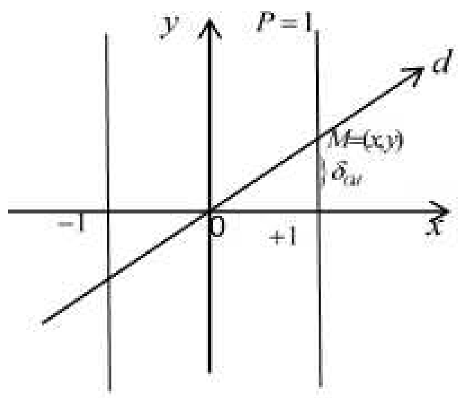

Fig. 2: Galilean unit circle

$$DH=\left\{\omega=z_1+\varepsilon z_2 \mid z_1,z_2\in H \text{ and } \varepsilon^2=0,\ \varepsilon\neq 0\right\}.$$

If we consider hyperbolic numbers $z_1=x_1+jx_2$ and $z_2=x_3+jx_4$, we represent a dual-hyperbolic number

$$\omega=x_1+x_2j+x_3\varepsilon+x_4\varepsilon j.$$

Here j, ε and εj are unit vectors in three-dimensional vectors space such that j is a hyperbolic unit, ε is a dual unit, and εj is a dual-hyperbolic unit [16]. So, the multiplication table of dual-hyperbolic numbers' base elements is given below.

$\times$	1	j	ε	$j\varepsilon$
1	1	j	ε	$j\varepsilon$
j	j	1	$j\varepsilon$	ε
ε	ε	$j\varepsilon$	0	0
$j\varepsilon$	$j\varepsilon$	ε	0	0

Table 1 Multiplication Table of Dual-Hyperbolic Numbers

We define addition and multiplication on dual-hyperbolic numbers as follows

$$\begin{array}{l}\omega_1+\omega_2=(z_1\pm\varepsilon z_2)+(z_3\pm\varepsilon z_4)=(z_1\pm z_3)+\varepsilon(z_2\pm z_4),\\ \omega_1\times\omega_2=(z_1+\varepsilon z_2)\times(z_3+\varepsilon z_4)=z_1z_3+\varepsilon(z_1z_4+z_2z_3)\end{array}$$

where ω_1 and ω_2 are dual-hyperbolic numbers and $z_1,z_2,z_3,z_4\in H$. On the other hand, the division of two dual-hyperbolic numbers is

$$\frac{\omega_1}{\omega_2}=\frac{z_1+\varepsilon z_2}{z_3+\varepsilon z_4}=\frac{z_1}{z_3}+\varepsilon\frac{z_2z_3-z_1z_4}{{z_3}^2},$$

where $\mathrm{Re}(\omega_2)\neq 0$.

Thus, dual-hyperbolic numbers yield a commutative ring whose characteristic is 0. If we consider both algebraic and geometric properties of dual-hyperbolic numbers, we define five possible conjugations of dual-hyperbolic numbers. These are

$$\begin{array}{ll}\omega^{\dagger_1}=\bar{z}_1+\varepsilon\bar{z}_2, & (\text{hyperbolic conjugation}),\\ \omega^{\dagger_2}=z_1-\varepsilon z_2, & (\text{dual conjugation}),\\ \omega^{\dagger_3}=\bar{z}_1-\varepsilon\bar{z}_2, & (\text{coupled conjugation}),\\ \omega^{\dagger_4}=\overline{z_1}\left(1-\varepsilon\frac{z_2}{z_1}\right) & (\omega\in DH-A),\quad (\text{dual}-\text{hyperbolic conjugation}),\\ \omega^{\dagger_5}=z_2-\varepsilon z_1, & (\text{anti}-\text{dual conjugation}),\end{array}$$

where "-" denotes the standard hyperbolic conjugation and the zero divisors of DH is defined by the set A [17].

In regards to these definitions, we give the following proposition for modules of dual-hyperbolic numbers.

Proposition 1. *Let $\omega=z_1+\varepsilon z_2$ be a dual-hyperbolic number. Then we write*

$$\begin{array}{l}|\omega|^2_{\dagger_1}=\omega\times\omega^{\dagger_1}=|z_1|^2+2\varepsilon\mathrm{Re}(z_1\bar{z}_2)\in D\\ |\omega|^2_{\dagger_2}=\omega\times\omega^{\dagger_2}=z_1^2\in H\\ |\omega|^2_{\dagger_3}=\omega\times\omega^{\dagger_3}=|z_1|^2-2j\varepsilon\mathrm{Im}(z_1\bar{z}_2)\in DH\\ |\omega|^2_{\dagger_4}=\omega\times\omega^{\dagger_4}=|z_1|^2\in R\ (\omega\in DH-A)\\ |\omega|^2_{\dagger_5}=\omega\times\omega^{\dagger_5}=z_1z_2+\varepsilon\left(z_2^2-z_1^2\right)\in DH\end{array}$$

[17].

3 De-Moivre and Euler formulae for Dual-Hyperbolic number

The exponential representation of a dual-hyperbolic number is $\omega=z_1\,e^{\frac{z_2}{z_1}\varepsilon}$, where $\omega=z_1+\varepsilon z_2\in DH$ is a dual-hyperbolic number and $(z_1\neq 0)$. The dual-hyperbolic angle $\frac{z_2}{z_1}$ is called the argument of dual-hyperbolic number and it is denoted by $\arg\omega=\frac{z_2}{z_1}=\varphi$ [17].

Theorem 1. *Let $\omega=z_1+\varepsilon z_2\in DH-A$ be a dual-hyperbolic number and φ be the principal argument of ω. Every dual-hyperbolic number can be written in the form of*

$$\begin{array}{l}w=z_1e^{\varepsilon\varphi}\\ \quad=z_1\left(\mathrm{cosg}(\varphi)+\varepsilon\mathrm{sing}(\varphi)\right)=\left\{\begin{array}{l}r\left(\cosh\varphi+j\sinh\varphi\right)\left(\mathrm{cosg}(\varphi)+\varepsilon\mathrm{sing}(\varphi)\right),\ |x_1|>|y_1|\\ r\left(\sinh\varphi+j\cosh\varphi\right)\left(\mathrm{cosg}(\varphi)+\varepsilon\mathrm{sing}(\varphi)\right),\ |y_1|>|x_1|\end{array}\right.\end{array}$$

such that $\mathrm{cosg}(\varphi)=1$ *and* $\mathrm{sing}(\varphi)=\varphi$.

Proof: The exponential representation of a dual-hyperbolic number $\omega = z_1 + \varepsilon z_2 \in DH - A$ is $\omega = z_1\, e^{\frac{z_2}{z_1}\varepsilon}$, where dual-hyperbolic number $\frac{z_2}{z_1}$ is the principal argument φ. Thus, if we write ω in the form of

$$\omega = z_1 e^{\varepsilon\varphi} = z_1 \left(1 + \varepsilon\varphi + \frac{(\varepsilon\varphi)^2}{2!} + \frac{(\varepsilon\varphi)^3}{3!} + \dots\dots\dots\dots\dots\right)$$

from properties of the dual unit, we see that

$$\omega = z_1 e^{\varepsilon\varphi} = z_1\,(1 + \varepsilon\varphi) = z_1\,(\mathrm{cosg}(\varphi) + \varepsilon \mathrm{sing}(\varphi))\,.$$

Eventually, by considering each case of $|x_1| > |y_1|$ or $|y_1| > |x_1|$ if we substitute the hyperbolic number $z_1 = x_1 + j\,y_1 \in H$ into the last equation we get

$$\omega = \begin{cases} r\,(\cosh\varphi + j\sinh\varphi)\,(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))\,, & |x_1| > |y_1|\,, \\ r\,(\sinh\varphi + j\cosh\varphi)\,(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))\,, & |y_1| > |x_1|\,. \end{cases}$$

□

Theorem 2. *Let* $\omega = z_1 + \varepsilon z_2 \in DH - A$ *be a dual-hyperbolic number and* $\arg\omega = \frac{z_2}{z_1} = \varphi$*. Then* $\frac{1}{e^{\varepsilon\varphi}} = e^{\varepsilon(-\varphi)}$.

Proof: If we use the Euler formula for $\frac{1}{e^{\varepsilon\varphi}}$, we have

$$\begin{aligned} \tfrac{1}{e^{\varepsilon\varphi}} &= \tfrac{1}{\left(1+\varepsilon\varphi+\frac{(\varepsilon\varphi)^2}{2!}+\frac{(\varepsilon\varphi)^3}{3!}+\dots\dots\dots\dots\dots\right)} \\ &= \tfrac{1}{\cos g(\varphi)+\varepsilon\sin g(\varphi)}. \end{aligned}$$

If we multiply both the numerator and the denominator of the last fraction by $\mathrm{cosg}(\varphi) - \varepsilon\mathrm{sing}(\varphi)$, we get

$$\begin{aligned} \tfrac{1}{e^{\varepsilon\varphi}} &= \tfrac{1}{\mathrm{cosg}(\varphi)+\varepsilon\mathrm{sing}(\varphi)}\tfrac{(\mathrm{cosg}(\varphi)-\varepsilon\mathrm{sing}(\varphi))}{(\mathrm{cosg}(\varphi)-\varepsilon\mathrm{sing}(\varphi))} \\ &= \tfrac{\mathrm{cosg}(\varphi)-\varepsilon\mathrm{sing}(\varphi)}{\mathrm{cosg}^2(\varphi)}. \end{aligned}$$

If we consider equality $\mathrm{cosg}^2(\varphi) = 1$, we have

$$\frac{1}{e^{\varepsilon\varphi}} = \mathrm{cosg}(\varphi) - \varepsilon\mathrm{sing}(\varphi).$$

This gives us the relation

$$\frac{1}{e^{\varepsilon\varphi}} = \mathrm{cosg}(\varphi) - \varepsilon\mathrm{sing}(\varphi) = \mathrm{cosg}(-\varphi) + \varepsilon\mathrm{sing}(-\varphi).$$

As a consequence, we get $\frac{1}{e^{\varepsilon\varphi}} = e^{\varepsilon(-\varphi)}$. □

Theorem 3. *Let* $\omega = z_1 + \varepsilon z_2 \in DH - A$ *be a dual-hyperbolic number and* $\omega = z_1 e^{\varepsilon\varphi} = z_1\,(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))$ *be its polar representation. Then, the equation*

$$\omega^n = \left(z_1 e^{\varepsilon\varphi}\right)^n = (z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))^n = {z_1}^n\,(\mathrm{cosg}(n\varphi) + \varepsilon\mathrm{sing}(n\varphi))$$

yields for all non-negative integers.

Proof: First, let's prove that de-Moivre formula is correct for $n \in N$. For this, under consideration the Galilean trigonometric identities, for $n = 2$ the dual-hyperbolic number $\omega = z_1 e^{\varepsilon\varphi} \in DH - A$ becomes

$$\begin{aligned} (z_1 e^{\varepsilon\varphi})^2 &= z_1\,(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))\,z_1\,(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi)) \\ &= {z_1}^2\left(\mathrm{cosg}^2(\varphi) + \varepsilon\,(\mathrm{cosg}(\varphi)\mathrm{sing}(\varphi) + \mathrm{sing}(\varphi)\mathrm{cosg}(\varphi))\right) \\ &= {z_1}^2\,(\mathrm{cosg}(2\varphi) + \varepsilon\mathrm{sing}(2\varphi))\,. \end{aligned}$$

Suppose that the equality is true for $n = k$, that is,

$$(z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))^k = {z_1}^k\,(\mathrm{cosg}(k\varphi) + \varepsilon\mathrm{sing}(k\varphi))\,.$$

Then for the case $n = k + 1$, we find

$$\begin{aligned} (z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))^{k+1} &= z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))^k\,(z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi)) \\ &= {z_1}^k\,(\mathrm{cosg}(k\varphi) + \varepsilon\mathrm{sing}(k\varphi))\,z_1\,(\mathrm{cosg}(k\varphi) + \varepsilon\mathrm{sing}(k\varphi)) \\ &= {z_1}^k\,(\mathrm{cosg}(k\varphi)\mathrm{cosg}(\varphi) + \varepsilon\,(\mathrm{cosg}(k\varphi)\mathrm{sing}(\varphi) + \mathrm{sing}(k\varphi)\mathrm{cosg}(\varphi))) \\ &= {z_1}^{k+1}\,(\mathrm{cosg}((k+1)\varphi) + \varepsilon\mathrm{sing}((k+1)\varphi))\,. \end{aligned}$$

Here $z_1^k = r^k(\cosh(k\varphi) + j\sinh(k\varphi))$ for $|x_1| > |y_1|$ and $r = |z_1| = \mp\sqrt{x_1{}^2 - y_1{}^2}$. Moreover, $z_1^k = r^k(\sinh(k\varphi) + j\cosh(k\varphi))$ for $|y_1| > |x_1|$ and $r = |z_1| = \mp\sqrt{y_1{}^2 - x_1{}^2}$. On the other hand, for $\omega = z_1 e^{\varepsilon\varphi} \in DH - A$ and $n \in N$ we can write

$$\begin{aligned} w^{-n} &= z_1{}^{-n}(\mathrm{cosg}(n\varphi) - \varepsilon\mathrm{sing}(n\varphi)) \\ &= z_1{}^{-n}(\mathrm{cosg}(-n\varphi) + \varepsilon\mathrm{sing}(-n\varphi)). \end{aligned}$$

Thus, for all $n \in Z$ we obtain

$$\omega^n = \left(z_1 e^{\varepsilon\varphi}\right)^n = (z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))^n = z_1{}^n(\mathrm{cosg}(n\varphi) + \varepsilon\mathrm{sing}(n\varphi)).$$

□

Theorem 4. *The n-th degree root of ω is*

$$\sqrt[n]{\omega} = \sqrt[n]{z}\left(\mathrm{cosg}\left(\frac{\varphi}{n}\right) + \varepsilon\mathrm{sing}\left(\frac{\varphi}{n}\right)\right)$$

where $\omega = z_1 + \varepsilon z_2 \in DH - A$ is a dual-hyperbolic number.

Proof: Polar representation of $\omega = z_1 + \varepsilon z_2 \in DH - A$ is $\omega = z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))$. From Theorem 3, we know that

$$\omega^n = \left(z_1 e^{\varepsilon\varphi}\right)^n = (z_1(\mathrm{cosg}(\varphi) + \varepsilon\mathrm{sing}(\varphi))^n = z_1{}^n(\mathrm{cosg}(n\varphi) + \varepsilon\mathrm{sing}(n\varphi)).$$

So, we get

$$\begin{aligned} \sqrt[n]{\omega} &= \omega^{\frac{1}{n}} = z_1^{\frac{1}{n}}\left(\mathrm{cosg}\left(\tfrac{1}{n}\varphi\right) + \varepsilon\mathrm{sing}\left(\tfrac{1}{n}\varphi\right)\right) \\ &= \sqrt[n]{z_1}\left(\mathrm{cosg}\left(\tfrac{\varphi}{n}\right) + \varepsilon\mathrm{sing}\left(\tfrac{\varphi}{n}\right)\right). \end{aligned}$$

This completes the proof. □

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Compact Operators in the Class $\left(bv_k^\theta, bv\right)$

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Abstract: The space bv of bounded variation sequence plays an important role in the summability. More recently this space has been generalized to the space bv_k^θ and the class $\left(bv_k^\theta, bv\right)$ of infinite matrices has been characterized by Hazar and Sarıgöl [2]. In the present paper, for $1 < k < \infty$, we give necessary and sufficient conditions for a matrix in the same class to be compact, where θ is a sequence of positive numbers.

Keywords: Matrix transformations, Sequence spaces, bv_k^θ spaces.

1 Introduction

Let ω be the set of all complex sequences, ℓ_k and c be the set of k-absolutely convergent series and convergent sequences. In [2] , the space bv_k^θ has been defined by

$$bv_k^\theta = \left\{ x = (x_k) \in w : \sum_{n=0}^{\infty} \theta_n^{k-1} \left|\triangle x_n\right|^k < \infty,\ x_{-1} = 0 \right\},$$

which is a BK space for $1 \le k < \infty$, where (θ_n) is a sequence of nonnegative terms and $\triangle x_n = x_n - x_{n-1}$ for all n.

Also, in the special case $\theta_n = 1$ for all n, it is reduced to bv^k, studied by Malkowsky, Rakočević and Živković [1] , and $bv_1^\theta = bv$.

Let U and V be subspaces of w and $A = (a_{nv})$ be an arbitrary infinite matrix of complex numbers. By $A(x) = (A_n(x))$, we denote the A-transform of the sequence $x = (x_v)$, i.e.,

$$A_n(x) = \sum_{v=0}^{\infty} a_{nv} x_v,$$

provided that the series are convergent for $v, n \ge 0$. Then, A defines a matrix transformation from U into V, denoted by $A \in (U, V)$, if the sequence $Ax = (A_n(x)) \in V$ for all sequence $x \in U$.

Lemma 1.1 ([6]) . Let $1 < k < \infty$ and $1/k + 1/k^* = 1$. Then, $A \in (\ell_k, \ell)$ if and only if

$$\|A\|'_{(\ell_k,\ell)} = \left\{ \sum_{\nu=0}^{\infty} \left(\sum_{n=0}^{\infty} |a_{nv}| \right)^{k^*} \right\}^{1/k^*} < \infty$$

and there exists $1 \le \xi \le 4$ such that $\|A\|'_{(\ell_k,\ell)} = \xi \|A\|_{(\ell_k,\ell)}$

If S and H are subsets of a metric space (X, d) and $\varepsilon > 0$, then S is called an ε-net of H , if, for every $h \in H$, there exists an $s \in S$ such that $d(h, s) < \varepsilon$; if S is finite, then the ε-net S of H is called a finite $\varepsilon\text{-}net$ of H. By M_X, we denote the collection of all bounded subsets of X. If $Q \in M_X$, then the Hausdorff measure of noncompactness of Q is defined by

$$\chi(Q) = \inf\{\varepsilon > 0 : Q \text{ has a finite } \varepsilon\text{-net in } X\}.$$

The function $\chi : M_X \to [0, \infty)$ is called the Hausdorff measure of noncompactness [5].

If X and Y are normed spaces, $\mathcal{B}(X, Y)$ states the set of all bounded linear operators from X to Y and is also a normed space according to the norm $\|L\| = \sup_{x \in S_X} \|L(x)\|$, where S_X is a unit sphere in X, $i.e.$, $S_X = \{x \in X : \|x\| = 1\}$. Further, a lineer operator $L : X \to Y$ is said to be compact if the sequence $(L(x_n))$ has convergent subsequence in Y for every bounded sequence $x = (x_n) \in X$. By $\mathcal{C}(X, Y)$ we denote the set of such operators.

The following results are need to compute Hausdorff measure of noncompactness.

Lemma 1.2 ([4]). Let X and Y be Banach spaces, $L \in \mathcal{B}(X,Y)$. Then, Hausdorff measure of noncompactness of L, denoted by $\|L\|_{\chi}$, is defined by

$$\|L\|_{\chi} = \chi\left(L\left(S_X\right)\right),$$

and

$$L \in \mathcal{C}(X,Y) \text{ iff } \|L\|_{\chi} = 0.$$

Lemma 1.3 ([5]). Let Q be a bounded subset of the normed space X where $X = \ell_k$ for $1 \leq k < \infty$.If $P_r : X \to X$ is the operator defined by $P_r(x) = (x_0, x_1, ..., x_r, 0, ...)$ for all $x \in X$, then

$$\chi(Q) = \lim_{r\to\infty} \sup_{x\in Q} \left\|\left(I - P_r\right)(x)\right\|,$$

where I is the identity operator on X.

Lemma 1.4 ([4]). Let X be normed sequence space, χ_T and χ denote Hausdorff measures of noncompactness on M_{X_T} and M_X, the collections of all bounded sets in X_T and X, respectively. Then,

$$\chi_T(Q) = \chi(T(Q)) \text{ for all } Q \in M_{X_T},$$

where T is an infinite triangle matrix.

2 Compact operators on the space bv_k^θ

More recently the class $\left(bv_k^\theta, bv\right)$, $1 < k < \infty$, has been characterized by Hazar and Sarıgöl [2] in the following form. In the present paper, by computing Hausdorff measure of noncompactness, we characterize compact operators in the same class.

Theorem 2.1. Let $A = (a_{nv})$ be an infinite matrix of complex numbers for all $n, v \geq 0$ and $1 < k < \infty$. Then, $A \in \left(bv_k^\theta, bv\right)$ if and only if

$$\lim_{n\to\infty} \sum_{j=\nu}^{\infty} a_{nj} \text{ exists for each } v \tag{2.1}$$

$$\sup_m \sum_{\nu=0}^{m} \left|\theta_\nu^{-1/k^*} \sum_{j=\nu}^{m} a_{nj}\right|^{k^*} < \infty \text{ for each } n \tag{2.2}$$

$$\sum_{\nu=0}^{\infty} \left(\sum_{n=0}^{\infty} \left|\theta_\nu^{1/k^*} \sum_{j=\nu}^{\infty} \left(a_{nj} - a_{n-1,j}\right)\right|\right)^{k^*} < \infty. \tag{2.3}$$

Also, for special case $\theta_v = 1$, it is reduced to the following result of [1].

Corollary 2.2. Let $A = (a_{nv})$ be an infinite matrix of complex numbers for all $n, v \geq 0$ and $1 < k < \infty$. Then, $A \in \left(bv^k, bv\right)$ if and only if (2.1) holds,

$$\sup_m \sum_{\nu=0}^{m} \left|\sum_{j=\nu}^{m} a_{nj}\right|^{k^*} < \infty \text{ for each } n,$$

$$\sum_{\nu=0}^{\infty} \left(\sum_{n=0}^{\infty} \left|\sum_{j=\nu}^{\infty} \left(a_{nj} - a_{n-1,j}\right)\right|\right)^{k^*} < \infty.$$

Now we give the following theorem.

Theorem 2.3. Let $1 < k < \infty$ and $\theta = (\theta_n)$ be a sequence of positive numbers. If $A \in \left(bv_k^\theta, bv\right)$, then there exists $1 \leq \xi \leq 4$ such that

$$\|A\|_\chi = \frac{1}{\xi} \lim_{r\to\infty} \left\{ \sum_{n=r+1}^{\infty} \left(\sum_{v=0}^{\infty} |d_{nv}| \right)^{k^*} \right\}^{1/k^*}, \tag{2.4}$$

and $A \in \mathcal{C}\left(bv_k^\theta, bv\right)$ if and only if

$$\lim_{r\to\infty} \sum_{n=r+1}^{\infty} \left(\sum_{v=0}^{\infty} |d_{nv}| \right)^{k^*} = 0 \tag{2.5}$$

where

$$d_{nj} = \theta_j^{-1/k^*} \sum_{v=j}^{\infty} \left(a_{nv} - a_{n-1,v}\right)$$

Proof. Define $T_1 : bv_k^\theta \to \ell_k$ and $T_2 : bv \to \ell$ by $T_1(x) = \theta_v^{1/k^*}(x_v - x_{v-1})$ and $T_2(x) = x_v - x_{v-1},\ x_{-1} = 0$. Then, it clear that T_1 and T_2 are isomorhism preseving norms, $i.e.$, $\|x\|_{bv_k^\theta} = \|T_1(x)\|_{\ell_k}$ and $\|x\|_{bv} = \|T_2(x)\|_\ell$. So, bv_k^θ and bv are isometrically isomorhic to ℓ_k and ℓ, respectively, $i.e.$, $bv_k^\theta \simeq \ell_k$ and $bv \simeq \ell$. Now let $T_1(x) = y$ for $x \in bv_k^\theta$. Then, $x = T_1^{-1}(y) \in S_{bv_k^\theta}$ if and only if $y \in S_{\ell_k}$, where $S_X = \left\{x \in X : \|x\|_X = 1\right\}$. Also, it is seen easily (see [3]) that $T_2 A T_1^{-1} = D$ and $A \in \left(bv_k^\theta, bv\right)$ iff $D \in (\ell_k, \ell)$. Further, by Lemma 1.1, there exists $1 \leq \xi \leq 4$ such that

$$\begin{aligned}
\|A\|_{(bv_k^\theta, bv)} &= \sup_{x\neq\theta} \frac{\|A(x)\|_{bv}}{\|x\|_{bv_k^\theta}} = \sup_{x\neq\theta} \frac{\left\|T_2^{-1} D T_1(x)\right\|_{bv}}{\|x\|_{bv_k^\theta}} \\
&= \sup_{x\neq\theta} \frac{\|D(y)\|_\ell}{\|y\|_{\ell_k}} = \|D\|_{(\ell_k,\ell)} \\
&= \frac{1}{\xi} \|D\|'_{(\ell_k,\ell)}
\end{aligned}$$

and so, by Lemmas 1.2, 1.3 and 1.4, we have

$$\begin{aligned}
\|A\|_\chi &= \chi\left(AS_{bv_k^\theta}\right) = \chi(T_2 A S_{bv_k^\theta}) \\
&= \chi(D T_1 S_{bv_k^\theta}) = \lim_{r\to\infty} \sup_{y\in S_{\ell_k}} \|(I - P_r)\, D(y)\|_\ell \\
&= \lim_{r\to\infty} \sup_{y\in S_{\ell_k}} \left\|D^{(r)}(y)\right\| = \lim_{r\to\infty} \left\|D^{(r)}\right\|_{(\ell_k,\ell)} \\
&= \frac{1}{\xi} \lim_{r\to\infty} \left\{ \sum_{n=r+1}^{\infty} \left(\sum_{v=0}^{\infty} |d_{nv}| \right)^{k^*} \right\}^{1/k^*}
\end{aligned}$$

where $P_r : \ell \to \ell$ is defined by $P_r(y) = (y_0, y_1, ..., y_r, 0, ...)$, and

$$d_{nv}^{(r)} = \begin{cases} 0, & 0 \leq n \leq r \\ d_{nv}, & n > r \end{cases}$$

So the proof is completed by Lemma 1.2.

In the special case $\theta_n = 1$, the following result is immediate.

Corollary 2.4. Let $1 < k < \infty$. If $A \in \left(bv^k, bv\right)$, then there exists $1 \leq \xi \leq 4$ such that

$$\|A\|_\chi = \frac{1}{\xi} \lim_{r\to\infty} \left\{ \sum_{n=r+1}^{\infty} \left(\sum_{v=0}^{\infty} |d_{nv}| \right)^{k^*} \right\}^{1/k^*}$$

and

$$A \in \mathcal{C}\left(bv_k, bv\right) \text{ iff } \lim_{r\to\infty} \sum_{n=r+1}^{\infty} \left(\sum_{v=0}^{\infty} |d_{nv}| \right)^{k^*} = 0$$

where

$$d_{nj} = \sum_{v=j}^{\infty} \left(a_{nv} - a_{n-1,v}\right)$$

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Deferred Statistical Convergence in Metric Spaces

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Abstract: In this paper, the concept of deferred statistical convergence is generalized to general metric spaces, and some inclusion relations between deferred strong Cesàro summability and deferred statistical convergence are given in general metric spaces.

Keywords: Metric space, Statistical convergence, Deferred statistical convergence.

1 Introduction

The idea of statistical convergence was given by Zygmund [1] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [2] and Fast [3] and then reintroduced independently by Schoenberg [4]. Over the years and under different names, statistical convergence has been discussed in the Theory of Fourier Analysis, Ergodic Theory, Number Theory, Measure Theory, Trigonometric Series, Turnpike Theory and Banach Spaces. Later on it was further investigated from the sequence spaces point of view and linked with summability theory by Gupta and Bhardwaj [5], Braha et al. [6], Çınar et al. [7], Connor [8], Et et al. ([9],[10],[11],[12],[13]), Fridy [14], Işık et al. ([15],[16],[17]), Mohiuddine et al. [18], Mursaleen et al. [19], Nuray [20], Nuray and Aydın [21], Salat [22], Şengül et al. ([23],[24],[25],[26]), Srivastava et al. ([27],[28]) and many others.

The idea of statistical convergence depends upon the density of subsets of the set $\mathbb{N}$ of natural numbers. The density of a subset $\mathbb{E}$ of $\mathbb{N}$ is defined by

$$\delta(\mathbb{E}) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_{\mathbb{E}}(k),$$

provided that the limit exists, where $\chi_{\mathbb{E}}$ is the characteristic function of the set $\mathbb{E}$. It is clear that any finite subset of $\mathbb{N}$ has zero natural density and that

$$\delta\left(\mathbb{E}^c\right) = 1 - \delta\left(\mathbb{E}\right).$$

A sequence $x = (x_k)_{k\in\mathbb{N}}$ is said to be statistically convergent to L if, for every $\varepsilon > 0$, we have

$$\delta\left(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}\right) = 0.$$

In this case, we write

$$x_k \xrightarrow{\text{stat}} L \quad \text{as} \quad k \to \infty \qquad \text{or} \qquad S - \lim_{k\to\infty} x_k = L.$$

In 1932, Agnew [29] introduced the concept of deferred Cesàro mean of real (or complex) valued sequences $x = (x_k)$ defined by

$$(D_{p,q}x)_n = \frac{1}{(q(n) - p(n))} \sum_{k=p(n)+1}^{q(n)} x_k,\ n = 1, 2, 3, \ldots,$$

where $p = \{p(n)\}$ and $q = \{q(n)\}$ are the sequences of non-negative integers satisfying

$$p(n) < q(n) \text{ and } \lim_{n\to\infty} q(n) = \infty.$$

Let K be a subset of $\mathbb{N}$ and denote the set $\{k : p(n) < k \leq q(n), k \in K\}$ by $K_{p,q}(n)$.

Deferred density of K is defined by

$$\delta_{p,q}(K) = \lim_{n\to\infty} \frac{1}{(q(n) - p(n))} |K_{p,q}(n)|, \text{ provided the limit exists,}$$

where, vertical bars indicate the cardinality of the enclosed set $K_{p,q}(n)$. If $q(n) = n$, $p(n) = 0$, then the deferred density coincides with natural density of K.

A real valued sequence $x = (x_k)$ is said to be deferred statistically convergent to L, if for each $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{(q(n) - p(n))} |\{p(n) < k \leq q(n) : |x_k - L| \geq \varepsilon\}| = 0.$$

In this case we write $S_{p,q} - \lim x_k = L$. If $q(n) = n$, $p(n) = 0$, for all $n \in \mathbb{N}$, then deferred statistical convergence coincides with usual statistical convergence [30].

2 Main Results

In this section, we give some inclusion relations between statistical convergence, deferred strong Cesàro summability and deferred statistical convergence in general metric spaces.

Definition 1 Let (X, d) be a metric space and $\{p(n)\}$ and $\{q(n)\}$ be two sequences as above. A metric valued sequence $x = (x_k)$ is said to be $DS^d_{p,q}-$convergent (or deferred $d-$statistically convergent) to a if there is a real number $a \in X$ such that

$$\lim_{n\to\infty} \frac{1}{(q(n) - p(n))} |\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}| = 0.$$

In this case we write $DS^d_{p,q} - \lim x_k = a$ or $x_k \to a \left(DS^d_{p,q}\right)$. The set of all $DS^d_{p,q}-$statistically convergent sequences will be denoted by $DS^d_{p,q}$. If $q(n) = n$ and $p(n) = 0$, then deferred $d-$statistical convergence coincides $d-$statistical convergence.

Definition 2 Let (X, d) be a metric space and $\{p(n)\}$ and $\{q(n)\}$ be two sequences as above. A metric valued sequence $x = (x_k)$ is said to be strongly $Dw^d_{p,q}-$summable (or deferred strongly $d-$Cesàro summable) to a if there is a real number $a \in X$ such that

$$\lim_{n\to\infty} \frac{1}{(q(n) - p(n))} \sum_{p(n)+1}^{q(n)} d(x_k, a) = 0.$$

In this case we write $Dw^d_{p,q} - \lim x_k = a$ or $x_k \to a \left(Dw^d_{p,q}\right)$. The set of all strongly $Dw^d_{p,q}-$summable sequences will be denoted by $Dw^d_{p,q}$. If $q(n) = n$ and $p(n) = 0$, for all $n \in \mathbb{N}$, then deferred strong $d-$Cesàro summability coincides strong $d-$Cesàro summability.

Theorem 3 Let (X, d) be a linear metric space and $x = (x_k)$, $y = (y_k)$ be metric valued sequences, then

(i) If $DS^d_{p,q} - \lim x_k = x_0$ and $DS^d_{p,q} - \lim y_k = y_0$, then $DS^d_{p,q} - \lim(x_k + y_k) = x_0 + y_0$,

(ii)If $DS^d_{p,q} - \lim x_k = x_0$ and $c \in \mathbb{C}$, then $DS^d_{p,q} - \lim(cx_k) = cx_0$,

(iii) If $DS^d_{p,q} - \lim x_k = x_0$, $DS^d_{p,q} - \lim y_k = y_0$ and $x, y \in \ell_\infty$, then $DS^d_{p,q} - \lim(x_k y_k) = x_0 y_0$.

Theorem 4 $Dw^d_{p,q} \subseteq DS^d_{p,q}$ and the inclusion is strict.

Proof. First part of proof is easy, so omitted. To show the strictness of the inclusion, choose $q(n) = n, p(n) = 0$, for all $n \in \mathbb{N}$ and $a = 0$ and define a sequence $x = (x_k)$ by

$$x_k = \begin{cases} \frac{\sqrt{n}}{2}, & k = n^2 \\ 0, & k \neq n^2 \end{cases}.$$

Then for every $\varepsilon > 0$, we have

$$\frac{1}{(q(n) - p(n))} |\{p(n) < k \leq q(n) : d(x_k, 0) \geq \varepsilon\}| \leq \frac{[\sqrt{n}]}{n} \to 0, \text{ as } n \to \infty,$$

where $d(x, y) = |x - y|$, that is $x_k \to 0 \left(DS^d_{p,q}\right)$. At the same time, we get

$$\frac{1}{(q(n) - p(n))} \sum_{p(n)+1}^{q(n)} d(x_k, 0) \leq \frac{[\sqrt{n}]\,[\sqrt{n}]}{n} \to 1,$$

i.e. $x_k \nrightarrow 0 \left(Dw^d_{p,q}\right)$. Therefore, $Dw^d_{p,q} \subseteq DS^d_{p,q}$ is strict.

Theorem 5 If $\liminf_n \frac{q(n)}{p(n)} > 1$, then $S^d \subset DS^d_{p,q}$.

Proof. Suppose that $\liminf_n \frac{q(n)}{p(n)} > 1$; then there exists a $\nu > 0$ such that $\frac{q(n)}{p(n)} \geq 1 + \nu$ for sufficiently large n, which implies that

$$\frac{q(n) - p(n)}{q(n)} \geq \frac{\nu}{1+\nu} \Longrightarrow \frac{1}{q(n)} \geq \frac{\nu}{(1+\nu)} \frac{1}{(q(n) - p(n))}.$$

If $x_k \to a\left(S^d\right)$, then for every $\varepsilon > 0$ and for sufficiently large n, we have

$$\begin{aligned}\frac{1}{q(n)} \left|\{k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right| &\geq \frac{1}{q(n)} \left|\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right| \\ &\geq \frac{\nu}{(1+\nu)} \frac{1}{(q(n) - p(n))} \left|\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right|.\end{aligned}$$

This proves the proof.
"In the following theorem, by changing the conditions on the sequences (p_n) and (q_n) we give the same relation with Theorem 5."

Theorem 6 If $\lim_{n\to\infty} \inf \dfrac{(q(n) - p(n))}{n} > 0$ and $q(n) < n$, then $S^d \subseteq DS^d_{p,q}$.

Proof. Let $\lim_{n\to\infty} \inf \dfrac{(q(n) - p(n))}{n} > 0$ and $q(n) < n$, then for each $\varepsilon > 0$ the inclusion

$$\{k \leq n : d(x_k, a) \geq \varepsilon\} \supset \{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}$$

is satisfied and so we have the following inequality

$$\begin{aligned}\frac{1}{n} \left|\{k \leq n : d(x_k, a) \geq \varepsilon\}\right| &\geq \frac{1}{n} \left|\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right| \\ &= \frac{(q(n) - p(n))}{n} \frac{1}{(q(n) - p(n))} \left|\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right|.\end{aligned}$$

Therefore $S^d \subseteq DS^d_{p,q}$.

Theorem 7 Let $\{p(n)\}, \{q(n)\}, \{p'(n)\}$ and $\{q'(n)\}$ be four sequences of non-negative integers such that

$$p'(n) < p(n) < q(n) < q'(n) \text{ for all } n \in \mathbb{N}, \tag{1}$$

then

(i) If

$$\lim_{n\to\infty} \frac{q(n) - p(n)}{q'(n) - p'(n)} = m > 0 \tag{2}$$

then $DS^d_{p',q'} \subseteq DS^d_{p,q}$,

(ii) If

$$\lim_{n\to\infty} \frac{q'(n) - p'(n)}{q(n) - p(n)} = 1 \tag{3}$$

then $DS^d_{p,q} \subseteq DS^d_{p',q'}$.

Proof. (i) Let (2) be satisfied. For given $\varepsilon > 0$ we have

$$\{p'(n) < k \leq q'(n) : d(x_k, a) \geq \varepsilon\} \supseteq \{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\},$$

and so

$$\begin{aligned}&\frac{1}{(q'(n) - p'(n))} \left|\{p'(n) < k \leq q'(n) : d(x_k, a) \geq \varepsilon\}\right| \\ &\geq \frac{(q(n) - p(n))}{(q'(n) - p'(n))} \frac{1}{(q(n) - p(n))} \left|\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right|.\end{aligned}$$

Therefore $DS^d_{p',q'} \subseteq DS^d_{p,q}$.

(ii) Omitted.

Theorem 8 Let $\{p(n)\}, \{q(n)\}, \{p'(n)\}$ and $\{q'(n)\}$be four sequences of non-negative integers defined as in (1).

(i) If (2) holds then $Dw^d_{p',q'} \subset Dw^d_{p,q}$,

(ii) If (3) holds and $x = (x_k)$ be a bounded sequence, then $Dw^d_{p,q} \subset Dw^d_{p',q'}$.

Proof. Omitted.

Theorem 9 Let $\{p(n)\}, \{q(n)\}, \{p'(n)\}$ and $\{q'(n)\}$ be four sequences of non-negative integers defined as in (1). Then

(i) Let (2) holds, if a sequence is strongly $Dw^d_{p',q'}-$summable to a, then it is $DS^d_{p,q}-$convergent to a,

(ii) Let (3) holds and $x = (x_k)$ be a bounded sequence, if a sequence is $DS^d_{p,q}-$convergent to a then it is strongly $Dw^d_{p',q'}-$summable to a.

Proof. (i) Omitted.

(ii) Suppose that $DS^d_{p,q} - \lim x_k = a$ and $(x_k) \in \ell_\infty$. Then there exists some $M > 0$ such that $d(x_k, a) < M$ for all k, then for every $\varepsilon > 0$ we may write

$$
\begin{aligned}
&\frac{1}{(q'(n) - p'(n))} \sum_{p'(n)+1}^{q'(n)} d(x_k, a) \\
&= \frac{1}{(q'(n) - p'(n))} \sum_{q(n)-p(n)+1}^{q'(n)-p'(n)} d(x_k, a) + \frac{1}{(q'(n) - p'(n))} \sum_{p(n)+1}^{q(n)} d(x_k, a) \\
&\leq \frac{\big(q'(n) - p'(n)\big) - \big(q(n) - p(n)\big)}{(q'(n) - p'(n))} M + \frac{1}{(q'(n) - p'(n))} \sum_{p(n)+1}^{q(n)} d(x_k, a) \\
&\leq \left(\frac{q'(n) - p'(n)}{q(n) - p(n)} - 1\right) M + \frac{1}{(q(n) - p(n))} \sum_{\substack{p(n)+1 \\ d(x_k,a) \geq \varepsilon}}^{q(n)} d(x_k, a) \\
&+ \frac{1}{(q(n) - p(n))} \sum_{\substack{p(n)+1 \\ d(x_k,a) < \varepsilon}}^{q(n)} d(x_k, a) \\
&\leq \left(\frac{q'(n) - p'(n)}{q(n) - p(n)} - 1\right) M + \frac{M}{(q(n) - p(n))} \left|\{p(n) < k \leq q(n) : d(x_k, a) \geq \varepsilon\}\right| \\
&+ \frac{q'(n) - p'(n)}{q(n) - p(n)} \varepsilon.
\end{aligned}
$$

This completes the proof.

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A New Type Generalized Difference Sequence Space $m(\phi, p)(\Delta_m^n)$

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Abstract: Let (ϕ_n) be a non-decreasing sequence of positive numbers such that $n\phi_{n+1} \le (n+1)\phi_n$ for all $n \in \mathbb{N}$. The class of all sequences (ϕ_n) is denoted by Φ. The sequence space $m(\phi)$ was introduced by Sargent [1] and he studied some of its properties and obtained some relations with the space ℓ_p. Later on it was investigated by Tripathy and Sen [2] and Tripathy and Mahanta [3]. In this work, using the generalized difference operator Δ_m^n, we generalize the sequence space $m(\phi)$ to sequence space $m(\phi, p)(\Delta_m^n)$, give some topological properties about this space and show that the space $m(\phi, p)(\Delta_m^n)$ is a $BK-$space by a suitable norm. The results obtained are generalizes some known results.

Keywords: Difference sequence, BK-space, Symmetric space, Normal space.

1 Introduction

By w, we denote the space of all complex (or real) sequences. If $x \in w$, then we simply write $x = (x_k)$ instead of $x = (x_k)_{k=0}^{\infty}$. We shall write ℓ_∞, c and c_0 for the sequence spaces of all bounded, convergent and null sequences, respectively. Also by ℓ_1 and ℓ_p; we denote the spaces of all absolutely summable and $p-$absolutely summable sequences, respectively.

Let $x \in w$ and $S(x)$ denotes the set of all permutation of the elements x_n, i.e. $S(x) = \{(x_{\pi(n)}) : \pi(n) \text{ is a permutation on } \mathbb{N}\}$. A sequence space E is said to be symmetric if $S(x) \subset E$ for all $x \in E$.

A sequence space E is said to be solid (normal) if $(y_n) \in E$, whenever $(x_n) \in E$ and $|y_n| \le |x_n|$ for all $n \in \mathbb{N}$.

A sequence space E is said to be sequence algebra if $x.y \in E$, whenever $x, y \in E$.
A sequence space E is said to be perfect if $E = E^{\alpha\alpha}$.
It is well known that if E is perfect then E is normal.

A sequence space E with a linear topology is called a $K-$space provided each of the maps $p_i : E \to \mathbb{C}$ defined by $p_i(x) = x_i$ is continuous for each $i \in \mathbb{N}$, where $\mathbb{C}$ denotes the complex field. A $K-$space E is called an $FK-$space provided E is a complete linear metric space. An $FK-$space whose topology is normable is called a $BK-$space.

The notion of difference sequence spaces was introduced by Kızmaz [4] and it was generalized by Et and Çolak [5] for $X = \ell_\infty, c, c_0$ as follows:

Let n be a non-negative integer, then

$$\Delta^n(X) = \{x = (x_k) : (\Delta^n x_k) \in X\},$$

where $\Delta^n x_k = \Delta^{n-1}x_k - \Delta^{n-1}x_{k+1}$ for all $k \in \mathbb{N}$ and so $\Delta^n x_k = \sum_{v=0}^{n} (-1)^v \binom{n}{v} x_{k+v}$. Et and Çolak [5] showed that the sequence spaces $\Delta^n(c_0)$, $\Delta^n(c)$ and $\Delta^n(\ell_\infty)$ are $BK-$spaces with the norm

$$\|x\|_{\Delta 1} = \sum_{i=1}^{n} |x_i| + \|\Delta^n x\|_\infty .$$

After then, using a new difference operator Δ_m^n, Tripathy et al. ([6], [7], [8]) have defined a new type difference sequence space $\Delta_m^n(X)$ such as

$$\Delta_m^n(X) = \{x = (x_k) : (\Delta_m^n x_k) \in X\},$$

where $m, n \in \mathbb{N}$, $\Delta_m^0 x = x$, $\Delta_m^1 x = (x_k - x_{k+m})$, $\Delta_m^n x = (\Delta_m^n x_k) = \left(\Delta_m^{n-1} x_k - \Delta_m^{n-1} x_{k+m}\right)$ and so $\Delta_m^n x_k = \sum_{v=0}^{n} (-1)^n \binom{n}{v} x_{k+mv}$, and give some topological properties about this space and show that the spaces $\Delta_m^n(X)$ are $BK-$spaces by the norm

$$\|x\|_{\Delta 2} = \sum_{i=1}^{mn} |x_i| + \|\Delta_m^n x\|_\infty$$

for $X = \ell_\infty, c$ and c_0. Recently, difference sequences have been studied in ([9],[10],[11],[12],[13],[14],[15],[16],[17],[18]) and many others.

2 Main results

In this section, we introduce a new class $m(\phi, p)(\Delta_m^n)$ of sequences , establish some inclusion relations and some topological properties. The obtained results are more general than those of Çolak and Et [19], Sargent [1] and Tripathy and Sen [2] .

The notation φ_s denotes the class of all subsets of $\mathbb{N}$, those do not contain more than s elements. Let (ϕ_n) be a non-decreasing sequence of positive numbers such that $n\phi_{n+1} \leq (n+1)\phi_n$ for all $n \in \mathbb{N}$. The class of all sequences (ϕ_n) is denoted by Φ.

The sequence spaces $m(\phi)$ and $m(\phi, p)$ were introduced by Sargent [1], Tripathy and Sen [2] as follows, respectively

$$m(\phi) = \left\{ x = (x_k) \in w : \|x\|_{m(\phi)} = \sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{k\in\sigma} |x_k| < \infty \right\},$$

$$m(\phi, p) = \left\{ x = (x_k) \in w : \|x\|_{m(\phi,p)} = \sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s} \left(\sum_{k\in\sigma} |x_k|^p \right)^{\frac{1}{p}} < \infty \right\}.$$

Let $m, n \in \mathbb{N}$ and $1 \leq p < \infty$. Now we define the sequence space $m(\phi, p)(\Delta_m^n)$ as

$$m(\phi, p)(\Delta_m^n) = \left\{ x = (x_k) \in w : \sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{k\in\sigma} \left|\Delta_m^n x_k\right|^p < \infty \right\}.$$

From this definition it is clear that $m(\phi, p)\left(\Delta_m^0\right) = m(\phi, p)$ and $m(\phi, 1)\left(\Delta_m^0\right) = m(\phi)$. In case of $m = 1$, we shall write $m(\phi, p)(\Delta^n)$ instead of $m(\phi, p)(\Delta_m^n)$ and in case of $p = 1$, we shall write $m(\phi)(\Delta_m^n)$ instead of $m(\phi, p)(\Delta_m^n)$. The sequence space $m(\phi, p)(\Delta_m^n)$ contains some unbounded sequences for $m, n \geq 1$. For example, the sequence $(x_k) = (k^n)$ is an element of $m(\phi, p)(\Delta_m^n)$ for $m = 1$, but is not an element of ℓ_∞.

Theorem 1. The space $m(\phi, p)(\Delta_m^n)$ is a Banach space with the norm

$$\|x\|_{\Delta_m^n} = \sum_{i=1}^{r} |x_i| + \sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s} \left(\sum_{k\in\sigma} \left|\Delta_m^n x_k\right|^p \right)^{\frac{1}{p}}, \quad 1 \leq p < \infty, \tag{1}$$

where $r = mn$ for $m \geq 1, n \geq 1$.

Proof. It is a routine verification that $m(\phi, p)(\Delta_m^n)$ is a normed linear space normed by (1) for $1 \leq p < \infty$. Let $\left(x^l\right)$ be a Cauchy sequence in $m(\phi, p)(\Delta_m^n)$, where $x^l = (x_k^l)_{k=1}^\infty = \left(x_1^l, x_2^l, ...\right) \in m(\phi, p)(\Delta_m^n)$, for each $l \in \mathbb{N}$. Then given $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$\left\|x^l - x^t\right\|_{\Delta_m^n} = \sum_{i=1}^{r} \left|x_i^l - x_i^t\right| + \sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s} \left(\sum_{k\in\sigma} \left|\Delta_m^n \left(x_k^l - x_k^t\right)\right|^p \right)^{\frac{1}{p}} < \varepsilon \tag{2}$$

for all $l, t > n_0$. Hence we obtain

$$\left|x_k^l - x_k^t\right| \to 0 \text{ as } l, t \to \infty, \text{ for each } k \in \mathbb{N}.$$

Therefore $(x_k^l)_{l=1}^\infty = \left(x_k^1, x_k^2, ...\right)$ is a Cauchy sequence in $\mathbb{C}$. Since $\mathbb{C}$ is complete, it is convergent, that is,

$$\lim_l x_k^l = x_k$$

for each $k \in \mathbb{N}$. Using these infinite limits $x_1, x_2, x_3, ...$ let us define the sequence $x = (x_k)$. We should show that $x \in m(\phi, p)(\Delta_m^n)$ and $\left(x^l\right) \to x$. Taking limit as $t \to \infty$ in (2) , we get

$$\left\|x^l - x\right\|_{\Delta_m^n} = \sum_{i=1}^{r} \left|x_i^l - x_i\right| + \sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s} \left(\sum_{k\in\sigma} \left|\Delta_m^n \left(x_k^l - x_k\right)\right|^p \right)^{\frac{1}{p}} < \varepsilon \tag{3}$$

for all $l \geq n_0$. This shows that $\left(x^l\right) \to x$ as $l \to \infty$. From (3) we also have

$$\sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n\left(x_k^l - x_k\right)\right|^p\right)^{\frac{1}{p}} < \varepsilon$$

for all $l \geq n_0$. Hence $x^l - x = \left(x_k^l - x_k\right)_k \in m(\phi,p)(\Delta_m^n)$. Since $x^l - x, x^l \in m(\phi,p)(\Delta_m^n)$ and $m(\phi,p)(\Delta_m^n)$ is a linear space, we have $x = x^l - \left(x^l - x\right) \in m(\phi,p)(\Delta_m^n)$. Therefore $m(\phi,p)(\Delta_m^n)$ is complete.

Theorem 2. The space $m(\phi,p)(\Delta_m^n)$ is a $BK-$space.

Proof. Omitted.

Theorem 3. [2] i) The space $m(\phi,p)$ is a symmetric space,
ii) The space $m(\phi,p)$ is a normal space.

Theorem 4. The sequence space $m(\phi,p)(\Delta_m^n)$ is not sequence algebra, is not solid and is not symmetric, for $m,n,p \geq 1$.

Proof. For the proof of the Theorem, consider the following examples:

Example 1. It is obvious that, if $x = \left(k^{n-2}\right)$, $y = \left(k^{n-2}\right)$ and $m = 1$, then $x, y \in m(\phi,p)(\Delta_m^n)$, but $x.y \notin m(\phi,p)(\Delta_m^n)$. Hence $m(\phi,p)(\Delta_m^n)$ is not a sequence algebra.

Example 2. It is obvious that, if $x = \left(k^{n-1}\right)$ and $m = 1$, then $x \in m(\phi,p)(\Delta_m^n)$, but $(\alpha_k x_k) \notin m(\phi,p)(\Delta_m^n)$ for $(\alpha_k) = \left((-1)^k\right)$. Hence $m(\phi,p)(\Delta_m^n)$ is not solid.

Example 3. Let us consider the sequence $x = \left(k^{n-1}\right)$. Then $x \in m(\phi,p)(\Delta_m^n)$ for $m = 1$. Let (y_k) be a rearrangement of (x_k) which is defined as follows:

$$y_k = \{x_1, x_2, x_4, x_3, x_9, x_5, x_{16}, x_6, x_{25}, x_7, x_{36}, x_8, x_{49}, x_{10}, ...\}.$$

Then $y \notin m(\phi,p)(\Delta_m^n)$. Hence $m(\phi,p)(\Delta_m^n)$ is not symmetric.

The following result is a consequence of Theorem 4.

Corollary 1. The sequence space $m(\phi,p)(\Delta_m^n)$ is not perfect, for $m,n,p \geq 1$.

Theorem 5. $m(\phi)(\Delta_m^n) \subset m(\phi,p)(\Delta_m^n)$ for each $m,n,p \geq 1$.

Proof. Omitted.

Theorem 6. $m(\phi,p)(\Delta_m^n) \subset m(\psi,p)(\Delta_m^n)$ if and only if $\sup\limits_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Suppose that $\sup\limits_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) < \infty$. Then $\phi_s \leq K\psi_s$ for every s and for some positive number K. If $x \in m(\phi,p)(\Delta_m^n)$, then,

$$\sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\phi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p\right)^{\frac{1}{p}} < \infty.$$

Now, we have

$$\sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\psi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p\right)^{\frac{1}{p}\%} < \sup_{s\geq 1}(K) \sup_{s\geq 1\quad k\in\varphi_s} \frac{1}{\phi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p\right)^{\frac{1}{p}} < \infty.$$

Hence $x \in m(\psi,p)(\Delta_m^n)$.

Conversely let $m(\phi,p)(\Delta_m^n) \subset m(\psi,p)(\Delta_m^n)$ and suppose that $\sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) = \infty$. Then, there exists a sequence (s_i) of natural numbers such that $\lim\limits_i\left(\frac{\phi_{s_i}}{\psi_{s_i}}\right) = \infty$. Then, for $x \in m(\phi,p)(\Delta_m^n)$ we have

$$\sup_{s\geq 1,\ \sigma\in\varphi_s} \frac{1}{\psi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p\right)^{\frac{1}{p}} \geq \sup_{i\geq 1}\left(\frac{\phi_{s_i}}{\psi_{s_i}}\right) \sup_{i\geq 1,\ \sigma\in\varphi_{s_i}} \frac{1}{\phi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p\right)^{\frac{1}{p}} = \infty.$$

Therefore $x \notin m(\psi,p)(\Delta_m^n)$. This contradict to $m(\phi,p)(\Delta_m^n) \subset m(\psi,p)(\Delta_m^n)$. Hence $\sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) < \infty$.

From Theorem 6, we get the following result.

Corollary 2. $m(\phi, p)(\Delta_m^n) = m(\psi, p)(\Delta_m^n)$ if and only if $0 < \inf_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) \leq \sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) < \infty$.

Theorem 7. $m(\phi, p)\left(\Delta_m^{n-1}\right) \subset m(\phi, p)(\Delta_m^n)$ and the inclusion is strict.

*Proof.*Let $x \in m(\phi, p)\left(\Delta_m^{n-1}\right)$. It is well known that, for $1 \leq p < \infty$, $|a+b|^p \leq 2^p\left(|a|^p + |b|^p\right)$. Hence, for $1 \leq p < \infty$, we have

$$\frac{1}{\phi_s}\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p \leq 2^p\left(\frac{1}{\phi_s}\sum_{k\in\sigma}\left|\Delta_m^{n-1}x_k\right|^p + \frac{1}{\phi_s}\sum_{k\in\sigma}\left|\Delta_m^{n-1}x_{k+1}\right|^p\right)$$

Hence $x \in m(\phi, p)(\Delta_m^n)$.

To show the inclusion is strict consider the following example.

Example 4. Let $\phi_n = 1$, for all $n \in \mathbb{N}$, $m = 1$ and $x = \left(k^{n-1}\right)$, then $x \in \ell_p(\Delta_m^n) \setminus \ell_p\left(\Delta_m^{n-1}\right)$.

Theorem 8. We have $\ell_p(\Delta_m^n) \subset m(\phi, p)(\Delta_m^n) \subset \ell_\infty(\Delta_m^n)$.

Proof. Since $m(\phi, p)(\Delta_m^n) = \ell_p(\Delta_m^n)$ for $\phi_n = 1$, for all $n \in \mathbb{N}$, then $\ell_p(\Delta_m^n) \subset m(\phi, p)(\Delta_m^n)$. Now assume that $x \in m(\phi, p)(\Delta_m^n)$. Then we have

$$\sup_{s\geq 1,\ \sigma\in\varphi_s}\frac{1}{\phi_s}\left(\sum_{k\in\sigma}\left|\Delta_m^n x_k\right|^p\right)^{\frac{1}{p}} < \infty \text{ and so } \left|\Delta_m^n x_k\right| < K\phi_1,$$

for all $k \in \mathbb{N}$ and for some positive number K. Thus, $x \in \ell_\infty(\Delta_m^n)$.

Theorem 9. If $0 < p < q$, then $m(\phi, p)(\Delta_m^n) \subset m(\phi, q)(\Delta_m^n)$.

Proof. Proof follows from the following inequality

$$\left(\sum_{k=1}^{n}|x_k|^q\right)^{\frac{1}{q}} \leq \left(\sum_{k=1}^{n}|x_k|^p\right)^{\frac{1}{p}}, \quad (0 < p < q).$$

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On Some Generalized Deferred Cesàro Means-II

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Abstract: In this study, using the genealized difference operator Δ^m, we introduce some new sequence spaces and investigate some topological properties of these sequence spaces
Keywords: Difference sequence, Deferred Cesaro mean.

1 Introduction

Let w be the set of all sequences of real or complex numbers and ℓ_∞, c and c_0 be respectively the Banach spaces of bounded, convergent and null sequences $x = (x_k)$ with the usual norm $\|x\|_\infty = \sup |x_k|$, where $k \in \mathbb{N} = \{1, 2, \ldots\}$, the set of positive integers. Also by bs, cs, ℓ_1 and ℓ_p; we denote the spaces of all bounded, convergent, absolutely summable and $p-$absolutely summable sequences, respectively.

A sequence space X with a linear topology is called a $K-$space provided each of the maps $p_i : X \to \mathbb{C}$ defined by $p_i(x) = x_i$ is continuous for each $i \in \mathbb{N}$, where $\mathbb{C}$ denotes the complex field. A $K-$space X is called an $FK-$space provided X is a complete linear metric space. An $FK-$space whose topology is normable is called a $BK-$space. We say that an $FK-$space X has AK (or has the AK property), if (e_k) (the sequence of unit vectors) is a Schauder bases for X.

The notion of difference sequence spaces was introduced by Kızmaz [**?**] and the notion was generalized by Et and Çolak [**?**]. Later on Et and Nuray [**?**] generalized these sequence spaces to the following sequence spaces:

Let X be any sequence space and let m be a non-negative integer. Then,

$$\Delta^m(X) = \{x = (x_k) : (\Delta^m x_k) \in X\}$$

$\Delta^0 x = (x_k)$, $\Delta^m x = \left(\Delta^{m-1}x_k - \Delta^{m-1}x_{k+1}\right)$ and so $\Delta^m x_k = \sum\limits_{i=0}^{m} (-1)^i \binom{m}{i} x_{k+i}$. is a Banach space normed by

$$\|x\|_\Delta = \sum_{i=1}^{m} |x_i| + \|\Delta^m x_k\|_\infty .$$

If $x \in X(\Delta^m)$ then there exists one and only one $y = (y_k) \in X$ such that

$$x_k = \sum_{i=1}^{k-m} (-1)^m \binom{k-i-1}{m-1} y_i = \sum_{i=1}^{k} (-1)^m \binom{k+m-i-1}{m-1} y_{i-m}, \quad y_{1-m} = y_{2-m} = \cdots = y_0 = 0$$

for sufficiently large k, for instance $k > 2m$. Recently, a large amount of work has been carried out by many mathematicians regarding various generalizations of sequence spaces. For a detailed account of sequence spaces one may refer to ([2-13]).

In 1932, Agnew [**?**] introduced the concept of deferred Cesaro mean of real (or complex) valued sequences $x = (x_k)$ defined by

$$(D_{p,q}x)_n = \frac{1}{(q(n) - p(n))} \sum_{k=p(n)+1}^{q(n)} x_k,\ n = 1, 2, 3, \ldots,$$

where $p = \{p(n)\}$ and $q = \{q(n)\}$ are the sequences of non-negative integers satisfying

$$p(n) < q(n) \text{ and } \lim_{n\to\infty} q(n) = \infty. \tag{1}$$

2 Topological Properties of $X(\Delta^m)$

In this section we prove some results involving the sequence spaces $C_0^d(\Delta^m), C_1^d(\Delta^m)$ and $C_\infty^d(\Delta^m)$.

Definition 1. *Let m be a fixed non-negative integer and let $\{p(n)\}$ and $\{q(n)\}$ be two sequences of non-negative integers satisfying the condition (1). We define the following sequence spaces:*

$$C_0^d(\Delta^m) = \left\{ x = (x_k) : \lim_n \frac{1}{(q(n) - p(n))} \sum_{k=p(n)+1}^{q(n)} \Delta^m x_k = 0 \right\},$$

$$C_1^d(\Delta^m) = \left\{ x = (x_k) : \lim_n \frac{1}{(q(n) - p(n))} \sum_{k=p(n)+1}^{q(n)} \left(\Delta^m x_k - L\right) = 0 \right\},$$

$$C_\infty^d(\Delta^m) = \left\{ x = (x_k) : \sup_n \left(\frac{1}{(q(n) - p(n))} \sum_{k=p(n)+1}^{q(n)} \Delta^m x_k \right) < \infty \right\}.$$

The above sequence spaces contain some unbounded sequences for $m \geq 1$, for example let $x = (k^m)$, then $x \in C_\infty^d(\Delta^m)$, but $x \notin \ell_\infty$.

Theorem 1. *The sequence spaces $C_0^d(\Delta^m), C_1^d(\Delta^m)$ and $C_\infty^d(\Delta^m)$ are Banach spaces normed by*

$$\|x\|_\Delta = \sum_{i=1}^{m} |x_i| + \sup_n \frac{1}{(q(n) - p(n))} \left| \sum_{k=p(n)+1}^{q(n)} \Delta^m x_k \right|.$$

Proof: Proof follows from Theorem **??** of Et and Nuray [**?**]. □

Theorem 2. *$X(\Delta^{m-1}) \subset X(\Delta^m)$ and the inclusion is strict for $X = C_0^d, C_1^d$ and C_∞^d.*

Proof: The inclusions part of the proof are esay. To see that the inclusions are strict, let $m = 2$ and $q(n) = n, p(n) = 0$ and consider a sequence defined by $x = \left(k^2\right)$, then $x \in C_1^d(\Delta^2)$, but $x \notin C_1^d(\Delta)$ (If $x = \left(k^2\right)$, then $\left(\Delta^2 x_k\right) = (2, 2, 2, ...)$. □

Theorem 3. *The inclusions $C_0^d(\Delta^m) \subset C_1^d(\Delta^m) \subset C_\infty^d(\Delta^m)$ are strict.*

Proof: First inclusion is esay. Second inclusion follows from the following inequality

$$\begin{aligned} \frac{1}{(q(n) - p(n))} \left| \sum_{k=p(n)+1}^{q(n)} \Delta^m x_k \right| &\leq \frac{1}{(q(n) - p(n))} \left| \sum_{k=p(n)+1}^{q(n)} \Delta^m x_k - L \right| + \frac{1}{(q(n) - p(n))} \left| \sum_{k=p(n)+1}^{q(n)} L \right| \\ &\leq \frac{1}{(q(n) - p(n))} \left| \sum_{k=p(n)+1}^{q(n)} \Delta^m x_k - L \right| + L \end{aligned}$$

For strict the inclusion, observe that $x = (1, 0, 1, 0, ...) \in C_\infty^d(\Delta^m)$, but $x \notin C_1^d(\Delta^m)$, (If $x = (1, 0, 1, 0, ...)$, then $(\Delta^m x_k) = \left((-1)^{m+1} 2^{m+1}\right)$). □

Theorem 4. *$C_1^d(\Delta^m)$ is a closed subspace of $C_\infty^d(\Delta^m)$.*

Proof: Proof follows from Theorem **??** of Et and Nuray [**?**]. □

Theorem 5. $C_1^d(\Delta^m)$ *is a nowhere dense subset of* $C_\infty^d(\Delta^m)$.

Proof: Proof follows from the fact that $C_1^d(\Delta^m)$ is a proper and complete subspace of $C_\infty^d(\Delta^m)$. □

Theorem 6. $C_\infty^d(\Delta^m)$ *is not separable, in general.*

Proof: Suppose that $C_\infty^d(\Delta^m)$ is separable for some $m \geq 1$, for example let $m = 2$ and $q(n) = n, p(n) = 0$. In this case $C_\infty(\Delta^2)$ is separable. In Theorem **??**, Bhardwaj et al. [**?**] show that $C_\infty(\Delta^2)$ is not separable. So $C_\infty^d(\Delta^m)$ is not separable, in general. □

Theorem 7. $C_\infty^d(\Delta^m)$ *does not have Schauder basis. separable, in general.*

Proof: Proof follows from the fact that if a normed space has a Schauder basis, then it is separable. □

Theorem 8. $C_1^d(\Delta^m)$ *is separable*.

Proof: Proof follows from Theorem **??** of Et and Nuray [**?**]. □

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Solutions of Singular Differential Equations by means of Discrete Fractional Analysis

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Abstract: Recently, many researchers demonstrated the usefulness of fractional calculus in the derivation of particular solutions of linear ordinary and partial differential equation of the second order. In this study, we acquire new discrete fractional solutions of singular differential equations (homogeneous and nonhomogeneous) by using discrete fractional nabla operator $\nabla^{\upsilon}(0<\upsilon<1)$.

Keywords: Discrete fractional analysis, Nabla operator, Singular differential equations.

1 Introduction

The remarkably widely investigated subject of fractional and discrete fractional calculus has gained importance and popularity during the past three decades or so, due chiefly to its demonstrated applications in numerous seemingly diverse fields of science and engineering [1]-[4]. The analogous theory for discrete fractional analysis was initiated and properties of the theory of fractional differences and sums were established. Recently, many articles related to discrete fractional analysis have been published [5]-[9]. The fractional nabla operator have been applied to various singular ordinary and partial differential equations such as the second-order linear ordinary differential equation of hypergeometric type [10], the Bessel equation [11], the Hermite equation [12], the non- fuchsian differential equation [13], the hydrogen atom equation [14].

The aim of this article is to obtain new dfs of the singular differential equation by means of fractional calculus operator.

2 Preliminary and properties

Here we only give a very short introduction to the basic definitions in discrete fractional calculus. For more on the subject we refer the reader to [5, 13].

Let $\zeta\in\mathbb{R}^{+}$, $n\in\mathbb{Z}$, such that $n-1\leq\zeta<n$. The $\zeta^{th}-$ order fractional sum of F is defined as

$$\nabla_c^{-\zeta}F(t)=\frac{1}{\Gamma(\zeta)}\sum_{\tau=c}^{t}(t-\rho(\tau))^{\overline{\zeta-1}}F(\tau), \tag{1}$$

where $t\in\mathbb{N}_\alpha=\{\alpha,\alpha+1,\alpha+2,...\}$, $\alpha\in\mathbb{R}$, $\rho(t)=t-1$ is the backward jump operator.

The rising factorial power and rising function is given by

$$t^{\overline{n}}=t(t+1)(t+2)...(t+n-1),\ n\in\mathbb{N},\ t^{\overline{0}}=1,$$

$$t^{\overline{\zeta}}=\frac{\Gamma(t+\zeta)}{\Gamma(t)},\ \zeta\in\mathbb{R},\ t\in\mathbb{R}\backslash\{...,-2,-1,0\},\ 0^{\overline{\zeta}}=0. \tag{2}$$

Note that

$$\nabla\left(t^{\overline{\zeta}}\right)=\zeta t^{\overline{\zeta-1}}, \tag{3}$$

where $\nabla\phi(t)=\phi(t)-\phi(\sigma(t))=\phi(t)-\phi(t-1)$.

The $\zeta^{th}-$ order fractional difference of F is defined by

$$\nabla_c^{\zeta}F(t)=\nabla^{n}\left[\nabla_c^{\zeta-(n-\zeta)}F(t)\right]$$

$$=\nabla^{n}\left[\frac{1}{\Gamma(n-\zeta)}\sum_{\tau=c}^{t}(t-\sigma(\tau))^{\overline{n-\zeta-1}}F(\tau)\right], \tag{4}$$

where F is defined on $\mathbb{N}_\alpha$.
Lemma 1. (Linearity). Let F and G be analytic and single-valued functions. Then

$$[c_1 F(t) + c_2 G(t)]_\zeta = c_1 F_\zeta(t) + c_2 G_\zeta(t), \tag{5}$$

where c_1 and c_2 are constants, $\zeta \in \mathbb{R};\ t \in \mathbb{C}$.
Lemma 2. (Index law). Let ϕ be an analytic and single-valued function. The following equality holds

$$\left(F_\zeta(t)\right)_\eta = F_{\zeta+\eta}(t) = \left(F_\eta(t)\right)_\zeta \ \left(F_\zeta(t) \neq 0;\ F_\eta(t) \neq 0;\ \zeta, \eta \in \mathbb{R};\ t \in \mathbb{C}\right). \tag{6}$$

Lemma 3. (Leibniz Rule). Suppose that F and G are analytic and single-valued functions. Then

$$\nabla_0^\zeta (FG)(t) = \sum_{n=0}^{t} \binom{\zeta}{n} \left[\nabla_0^{\zeta-n} F(t-n)\right] \left[\nabla^n G(t)\right],\ \zeta \in \mathbb{R};\ t \in \mathbb{C}, \tag{7}$$

where $\nabla^n G(t) = G_n(t)$ is the ordinary derivative of G of order $n \in \mathbb{N}_0$.
Definition 4. μ shift operator is given by

$$\mu^n F(t) = F(t-n) \tag{8}$$

where $n \in \mathbb{N}$.

3 Main results

Theorem 1. Let $F \in \{F : 0 \neq |F_\upsilon| < \infty;\ \upsilon \in \mathrm{R}\}$. Then the following homogeneous ordinary differential equation:

$$s(1-s)\ F_2 + [(\alpha - 2\gamma)\ s + \gamma + \sigma]\ F_1 + \gamma(\alpha - \gamma + 1)\ F = 0, \quad (s \in \mathrm{C}\backslash\{0, 1\}), \tag{9}$$

has particular solutions of the forms:

$$F = k\left\{s^{-(\upsilon\tau+\gamma+\sigma)}(1-s)^{-(\upsilon\tau+\gamma-\alpha-\sigma)}\right\}_{-(1+\upsilon)}, \tag{10}$$

and

$$F = ks^{1-(\gamma+\sigma)}\left\{s^{-(\upsilon\tau-\gamma-\sigma+2)}(1-s)^{-(\upsilon\tau+\gamma-\alpha-\sigma)}\right\}_{-(1+\upsilon)} \tag{11}$$

where $F_n = d^n F/ds^n\ (n = 0, 1, 2)$, $F_0 = F = F(s)$, $\alpha \neq 0$, γ, σ are given constants, k is an arbitrary constant and τ is a shift operator [15].
Proof. ***(i)*** When we operate ∇^υ to the both sides of (9), we readily obtain;

$$\nabla^\upsilon [F_2 s(1-s)] + \nabla^\upsilon \{F_1 [(\alpha - 2\gamma)\ s + \gamma + \sigma]\} + \nabla^\upsilon [F\gamma(\alpha - \gamma + 1)] = 0. \tag{12}$$

Using $(5) - (7)$ we have

$$\nabla^\upsilon [F_2 s(1-s)] = F_{2+\upsilon} s(1-s) + F_{1+\upsilon}\upsilon\tau(1-2s) - F_\upsilon \upsilon(\upsilon-1)\ \tau^2 \tag{13}$$

and

$$\nabla^\upsilon \{F_1 [(\alpha - 2\gamma)\ s + \gamma + \sigma]\} = F_{1+\upsilon} [(\alpha - 2\gamma)\ s + \gamma + \sigma] + F_\upsilon \upsilon\tau(\alpha - 2\gamma) \tag{14}$$

where τ is a shift operatÃűr. By substituting (13), (14) into the (12), we obtain

$$F_{2+\upsilon} s(1-s) + F_{1+\upsilon} [\upsilon\tau(1-2s) + (\alpha - 2\gamma)\ s + \gamma + \sigma]$$

$$+F_\upsilon \left[\upsilon(1-\upsilon)\ \tau^2 + \upsilon\tau(\alpha - 2\gamma) + \gamma(\alpha - \gamma + 1)\right] = 0. \tag{15}$$

Choose υ such that

$$\upsilon(1-\upsilon)\ \tau^2 + \upsilon\tau(\alpha - 2\gamma) + \gamma(\alpha - \gamma + 1) = 0,$$

$$\upsilon = \left[(\tau + \alpha - 2\gamma) \pm \sqrt{(\tau + \alpha - 2\gamma)^2 + 4\gamma(\alpha - \gamma + 1)}\right]/2\tau. \tag{16}$$

From Eq. (16), one can easily see that

$$\left[(\tau + \alpha - 2\gamma)^2 \geq 4\gamma(-\alpha + \gamma - 1)\right],$$

we have then

$$F_{2+\upsilon} s(1-s) + F_{1+\upsilon} [\upsilon\tau(1-2s) + (\alpha - 2\gamma)\ s + \gamma + \sigma] = 0, \tag{17}$$

from (15) and (16) .
Next, writing:

$$F_{1+\upsilon} = f(s) \quad \left[F = f_{-(1+\upsilon)}\right], \tag{18}$$

we have

$$f_1 + f\left[\frac{\upsilon\tau(1-2s) + (\alpha-2\gamma)\, s + \gamma + \sigma}{s(1-s)}\right] = 0, \tag{19}$$

from eqs. (17) and (18) . A particular solution of linear ordinary differential equation (19) :

$$f = ks^{-(\upsilon\tau+\gamma+\sigma)}(1-s)^{-(\upsilon\tau+\gamma-\alpha-\sigma)}. \tag{20}$$

Therefore, we obtain (10) from (18) and (20) .
(ii) Set

$$F = s^{\eta}\Phi, \quad \Phi = \Phi(s). \tag{21}$$

The first and second derivatives of (21) are acquired as follows:

$$F_1 = \eta s^{\eta-1}\Phi + s^{\eta}\Phi_1 \tag{22}$$

and

$$F_2 = \eta(\eta-1)\, s^{\eta-2}\Phi + 2\eta s^{\eta-1}\Phi_1 + s^{\eta}\Phi_2. \tag{23}$$

Substitute $(21) - (23)$ into (9) , we obtain

$$s(1-s)\, \Phi_2 + [(1-s)\, 2\eta + (\alpha-2\gamma)\, s + \gamma + \sigma]\, \Phi_1$$

$$+\left[\left(\left(\eta^2-\eta\right) + (\gamma+\sigma)\, \eta\right) s^{-1} - \left(\eta^2-\eta\right) + (\alpha-2\gamma)\, \eta + \gamma(\alpha-\gamma+1)\right] \Phi{=}0. \tag{24}$$

Choose η such that

$$\left(\eta^2-\eta\right) + (\gamma+\sigma)\, \eta = 0,$$

that is

$$\eta = 0, \quad \eta = 1-(\gamma+\sigma).$$

In the case $\eta = 0$, we have the same results as i.

Let $\eta = 1-(\gamma+\sigma)$. From (21) and (24) , we have

$$F = s^{1-(\gamma+\sigma)}\Phi \tag{25}$$

and

$$s(1-s)\, \Phi_2 + [(2\sigma+\alpha-2)\, s - (\gamma+\sigma-2)]\, \Phi_1 + [(1-\sigma)\, (\sigma+\alpha)]\, \Phi = 0 \tag{26}$$

respectively.
Applying the discrete operator ∇^{υ} to both sides of (26) , we obtain

$$\Phi_{2+\upsilon} s(1-s) + \Phi_{1+\upsilon}\left[\upsilon\tau(1-2s) + (2\sigma+\alpha-2)\, s - (\gamma+\sigma-2)\right]$$

$$+\Phi_{\upsilon}\left[\upsilon(1-\upsilon)\, \tau^2 + \upsilon\tau(2\sigma+\alpha-2) + (1-\sigma)\, (\alpha+\sigma)\right] {=}0. \tag{27}$$

Choose υ such that

$$\upsilon(1-\upsilon)\, \tau^2 + \upsilon\tau(2\sigma+\alpha-2) + (1-\sigma)\, (\alpha+\sigma) {=}0,$$

$$\upsilon = \left[(\tau+2\sigma+\alpha-2) \pm \sqrt{(\tau+2\sigma+\alpha-2)^2 - 4(\sigma-1)\, (\alpha+\sigma)}\right]/2\tau. \tag{28}$$

From Eq. (28), one can get

$$\left[(\tau+2\sigma+\alpha-2)^2 \geq 4(\sigma-1)\, (\alpha+\sigma)\right],$$

then we have

$$\Phi_{2+\upsilon} s(1-s) + \Phi_{1+\upsilon}\left[\upsilon\tau(1-2s) + (2\sigma+\alpha-2)\, s - (\gamma+\sigma-2)\right] = 0, \tag{29}$$

from (27) and (28).

Next, by writing

$$\Phi_{1+\upsilon} = g\,(s)\,, \quad \left[\Phi = g_{-(1+\upsilon)}\right], \tag{30}$$

we have

$$g_1 + g\left[\frac{\upsilon\tau\,(1-2s) + (2\sigma+\alpha-2)\;s - (\gamma+\sigma-2)}{s\,(1-s)}\right] = 0, \tag{31}$$

from (29) and (30) . A particular solution to this linear differential equation is given by

$$g = ks^{-(\upsilon\tau-\gamma-\sigma+2)}(1-s)^{-(\upsilon\tau+\gamma-\alpha-\sigma)}. \tag{32}$$

Thus we obtain the solution (11) from (25), (30) and (32).

4 Conclusion

In this article, we applied the nabla operator of discrete fractional analysis to the second order linear differential equations. We obtained the discrete fractional solutions of these equations via this new operator method.

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